Q32. Make a model of a straight staircase from a given rectangular strip of paper. What changes will you make in the design if the height is to be incresed with the same length of paper?

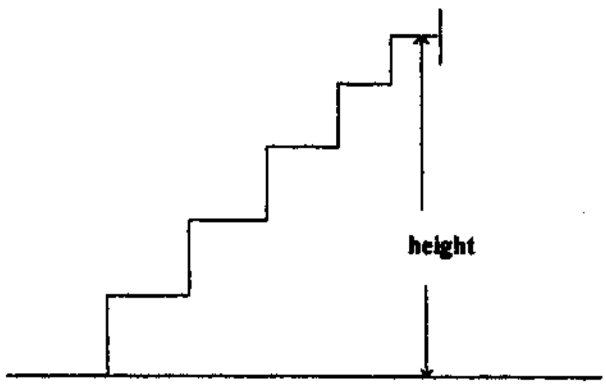


Fig : Q 32

Q 33. Make a model of a circular staircase from a given rectangular strip of paper.

Q34. Construct a proportionate human DNA with a long rectangular strip of a paper. The specifications of human DNA are as:

i. Width of a nucleotide pair = $20A^0$

ii. The distance between 2 nucleotide pairs = $3.4A^0$

iii. Number of nucleotide pairs per turn of the helix = 10

iv. Angle made by each nucleotide pair with the preceding pair = 36°

($A^0 = 10^{-8}$ cms)

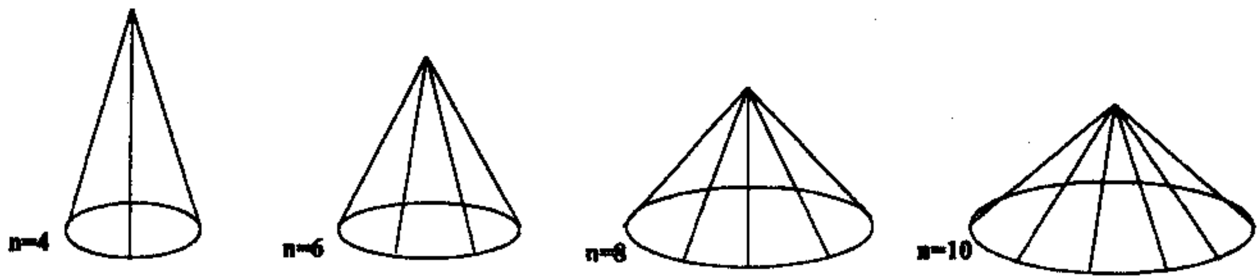


Fig: Q 28 (C) Cones made with different number of congruent sectors of a circle

Q29. Construct a cone with its base on top of right circular cylinder from a single square sheet.

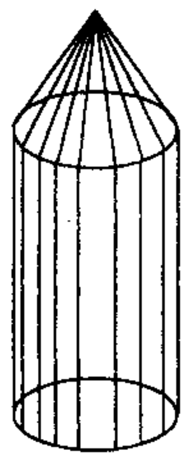


Fig: Q 29

Q30. Make a model of an oval shaped atheletic track from a single strip of paper. See that the model is symmetrical. Equations, measurement etc., are not necessary.

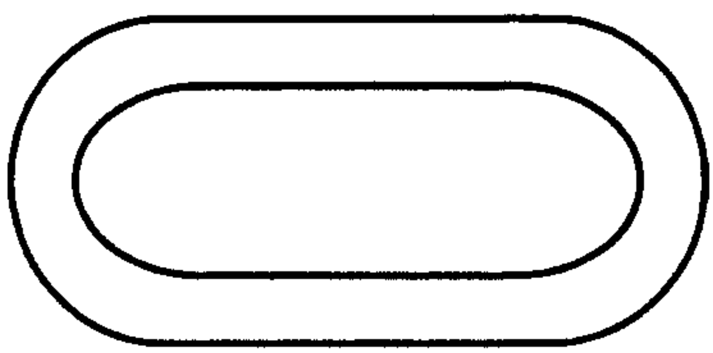


Fig : Q 30

Q31. Construct oval shaped model proportionate to following requirements:

a) mean straight path = 100x2m. (m-meter)

b) mean circular path = 75x2m.

c)width of the track = 5m.

Fig : Q 31

Q27. Construct a cone of height h=6 cms and radius r=4 cms. Calculate volume and surface area of this cone.

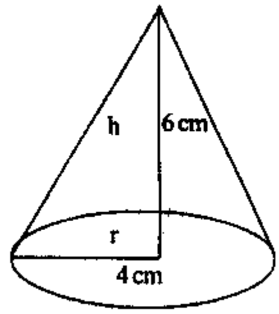


Fig: Q 27

Q28. Construct four cones with different number of divisions as shown in the table. Take four strips from a 20x20 cms square folded for 12 divisions rosette. Tabulate your readings as shown. If necessary increase the number of divisions.

When is the volume maximum?

Number of division used	C circumference in cms	r radius in cms	h height in cms	V volume in cm^3 s	A curved surface area in cm^2 s
4					
6					
8					
10					

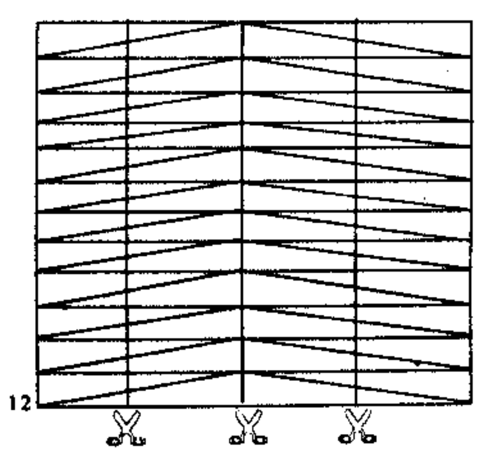


Fig: 28(QA) 12 divisions rosette square cut into 4 strips

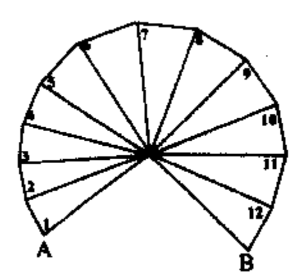


Fig: 28(QB) Cones made with different number of congruent sectors of a circle

Q24. Construct a right circular cylinder with a sleak fitting lid from two congruent square sheets. Write down the procedure in brief.

Q 25. Construct few cylinders of different diameters from 20x20 cms. square sheets and tabulate your readings in the following table. What is the ratio of height to width for maximum volume?

(This question is a "hands on experience" approach to question no. 21)

Width in cm.s	h height in cms.	A Base area in Cm^2s.	V Volume in cm^3s.	L lateral Surface area in cm^2s.	$\bar{A}$ Total Surface area A+L in cm^2s.
min.					
max					

Fig : Q 25

Q26. Make a set of close fitting cylinder and a cone. How will you prove from this set that volume of a cone equals 1/3 volume of a cylinder for the same height and circular base?

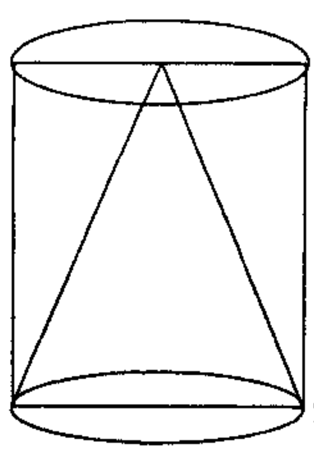


Fig : Q 26

Q19. Prove by origami method that lengths of tangent segments to a circle from an external point are equal and the two tangent segments are equally inclined to the line joining the point to the centre of circle.

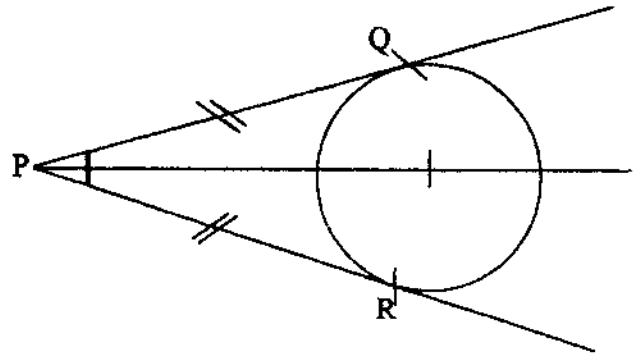


Fig: Q 19

Q20. Construct a rosette with full arc of angular measure 360° from a rectangular strip of paper. How will you construct a circular disc as shown in Fig. 20 B from a long narrow strip of paper.

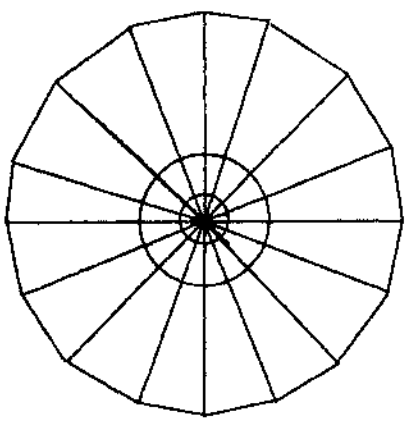


Fig: Q 20A Rosette with full arc of 360°

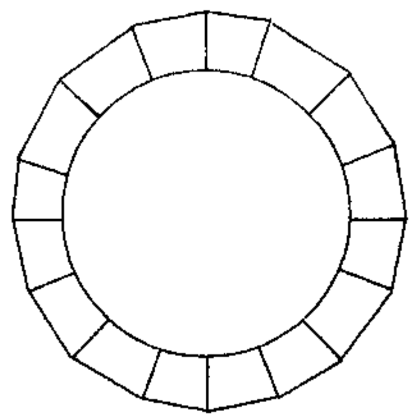


Fig: Q 20B Circular disc from a narrow strip of paper

Cylinders & Cones

Q21. Construct a cylinder with maximum volume from a given square. Prove by any method that the volume obtained is maximum.

Q22. Construct a shallow cylinder with usual fullscape rectangular sheet of paper (A4 size). Calculate the volume in millilitres.

Q23. Construct a cylinder with closed surfaces from a fullscape paper.

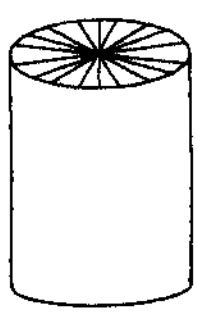


Fig : Q23

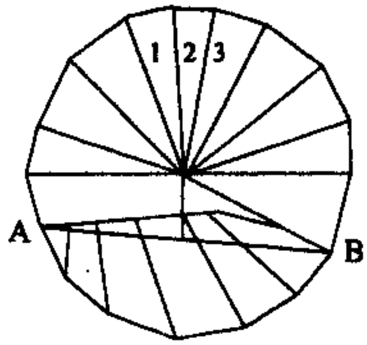
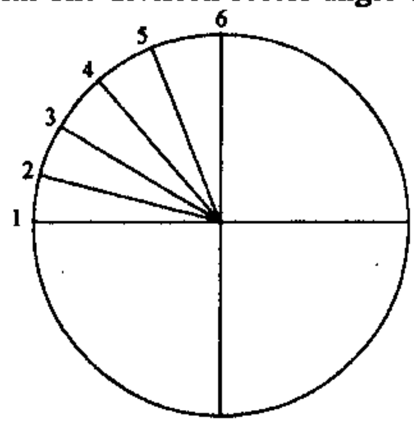


Fig: Q 14

Q 15. Construct a rosette with one division sector angle equal to 15^0 ($2\delta=15^0$)



($2\delta= 15^0$)

Fig: Q 15

Q16. Draw a sinusoidal waveform through 360^0 with help of the rosette constructed in question number 15.

Q 17. Calculate total angle subtended by a continuous side of a rosette (arc AB) at the centre for n=6,8,10&12. Does the sector angle increase with the increase in number of divisions? why?

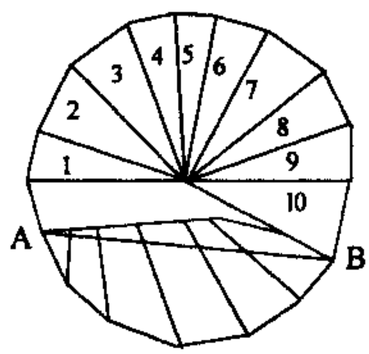


Fig: Q 17 Sector angle for arc AB for n= 10

Q18. Fold three equidistant concentric circles from a coloured rectangular strip

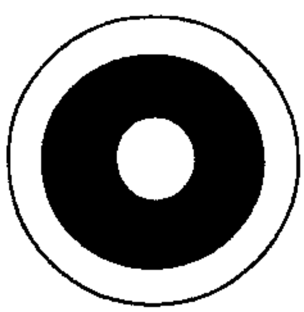


Fig: Q 18 Three equidistant concentric circles from a single strip

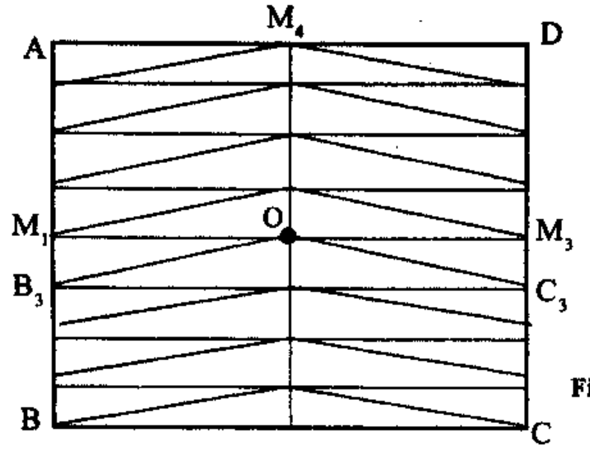


Fig: Q 11

Q12. State the number of axes of symmetry in the following figures.

Triangles:	**Quadrilaterals:**	**Pentagons:**
1. ΔM_4BC	5. Quad. ABCD	10. $M_4M_1BCM_3$
2. ΔM_3BC_2	6. Quad. ABM_2M_4	
3. ΔPBC	7. Quad. $ABCM_4$	**Hexagons**
4. ΔABC	8. Quad. ABC_3D_3	11. $M_4A_3B_3M_2C_3D_3$
	9. Quad. PABQ	

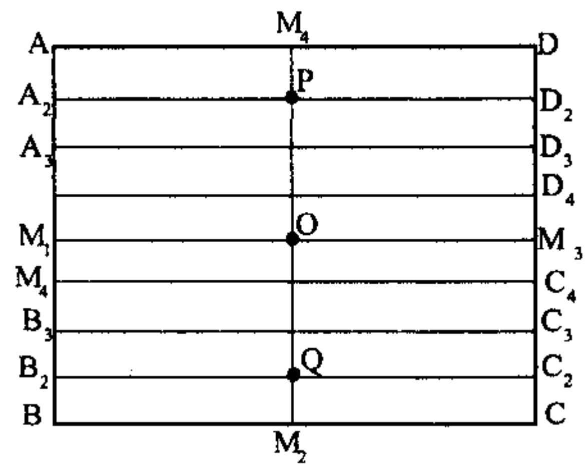


Fig: Q 12

Q13. Prove the Basic Proportionality Theorem (Thales Theorem) by origami method. (*Theorem: In a triangle a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio).*

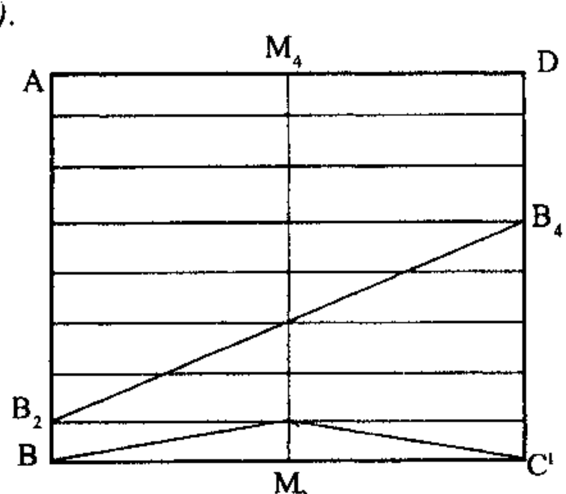


Fig: Q 13

Circles, Tangenets, Chords, Sectors etc.,

Q14. Make a rosette of 10 divisions from a 20 x 20 cms square and measure the following:

- i. Length of the chord AB
- ii. Distance of chord AB from the centre.
- iii. Lengths of major and minor arcsAB.
- iv. Angular measures of major and minor arcs AB.
- v. Area covered by a sector formed by 3 divisions & the sector angle.
- vi. Area of minor and major segments AB.

Triangles, Quadrilaterals & Symmetry

Q7. In a square of 8x8 units grid construct following right angled triangles with the sides given in the table. Calculate length of the side c in units from a paper gauze. Compare these values with the values obtained by using Pythogorean formula $a^2+b^2=c^2$.

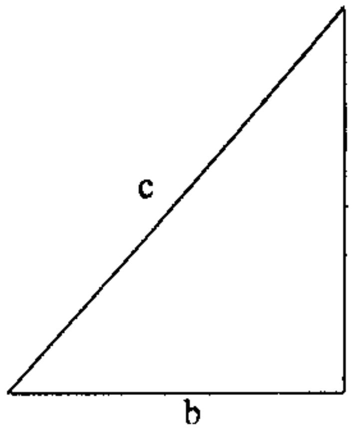


Fig: Q 7A.

a units	b units	c units
2	8	
4	8	
6	8	
8	8	

Fig: Q 7B.

Q8. In a square of 8x8 units construct isosceles right angled triangles with base = 2,4,6 and 8 units. What is the relation between base and height of an isosceles right angled triangle?

Q9. i) Construct an equilateral triangle PBC in a square ABCD.
ii) How will you construct an equilateral triangle with maximum area in the given square?
iii) How will you divide a given equilateral triangle into four equal equilateral triangles ?

(Clue for (ii) : Construct the required Δ symmetrically around the diagonal AC. And $\sin 30^0 = 1/2$)

Q10. Prove following theorems by origami method:

a) Two diagonals of a prallelogram bisect each other.

b) A quadrilateral is a parallaogram, if one pair of opposite sides are equal and parallel.

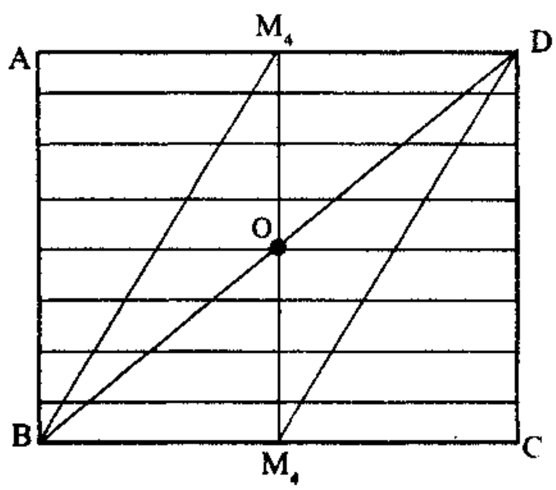


Fig: Q 10

Q11. Theorem on linear symmetry: Two triangls symmetrical with respect to a line are congruent. Prove this theorem by origami method.

Q5. Construct following angles on side BC of a square without using protractor or any other instrument. (i)14^0 (ii) 30^0 (iii) 31^0 (iv) 36^0 (v) 45^0 (vi)52^0 (vii) 76^0

(Clue $\tan 14^0 = 0.25$, $\sin 30^0 = 1/2$)

Q6. Take a big square and divide it equally into 8 horizontal and 4 vertical parts. Draw lines through point B and all intersecting point as shown in Fig.6A. Measure all angles by a protractor and tabulate your readings as shown in the table. What conclusions can be drawn from this table? Which angles are common in all? Can this exercise be used for introduction of trigonometric relations? What are the minimum and maximum angles?

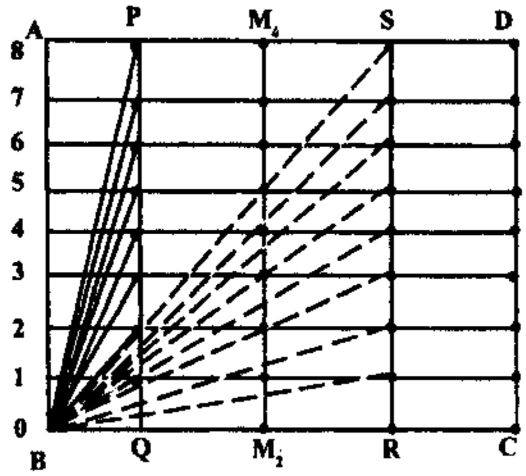


Fig: Q 6A. Angles made by 32 intersecting points

	Side PQ		Side M_4M_2		Side SR		Side DC	
Div. No.	Angle in degrees		Angle in degrees		Angle in degrees		Angle in degrees	
8	76							
7	74							
6	72							
5	68							
4	63							
3	56							
2	45				34			
1	27				9.5			

Fig: Q 6B. Table of angles made on each vertical side

Q3. Fold given squares so that each forms 100 and 121 equal squares. Where are such grids used?

Q4. Take two large size congruent squares. Mark seven equidistant points across AB as shown in Fig. Q4 A. Fold another square into a four layered rectangular strip with similar markings. Align the strip with side AB so that all corresponding points coincide. Now slide the strip downwards in steps of 1 unit. Observe that the corners A_1 and B_1 of the strip always lie on sides AB and BC respectively of the square. Mark positions of the points A_2, A_3.....B_2 for every step as shown in Fig Q4B. In the last step points A_1 and B_1 on strip will coincide with points B_1 and C on the square. All positions of points M_1 and B_3 are joined by smooth curves and shown in fig. 3. Draw curvs for other points also. What are these curves? Can you write down the equations for the same?

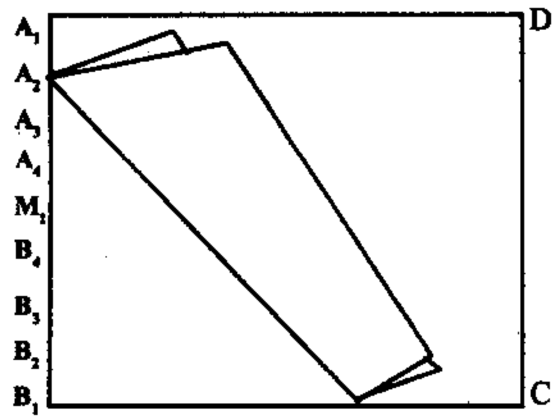


Fig:Q4A. Alignment of strip at A_2

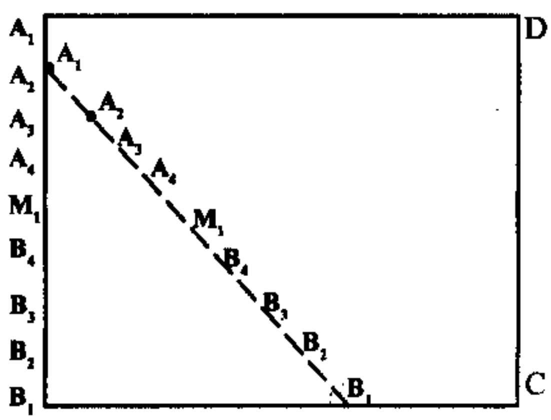


Fig:Q4B. Marking corresponding points on square

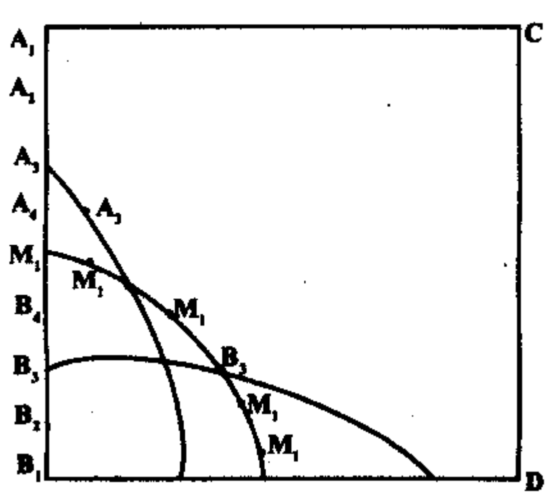


Fig:Q4C. Curves formed by joining all corresponding A_3, A_4 and M_1 points

Chapter - 36
Mathematics Exams Through Origami

If we imagine for a while that mathematics in schools is made simple and interesting through origami then `Mathematics Exams Through Origami' will be an obvious sequal of this method. Candidates will construct models according to questions and will flatten them again, retaining the creaes. Conventional answer books can also be used additionally for derivations, tables etc.

Material Required

a. Square papers of approximately 16x16 cms. & 20x20 cms.
b. Rectangular strips of required length from a roll of about 6 cms width.

Here are some examples of such questions based randomly from syllabus outlined in chapter 35. A detailed and rigorous questions bank can be worked out collectively with mathematics teachers and experts.

Parallel, Perpendicular, Inclined lines & Angles.

Q1. Make a grid of 16 equal squares as shown. Count the number of squares in this grid.

(**Ans.** 30 Squares)

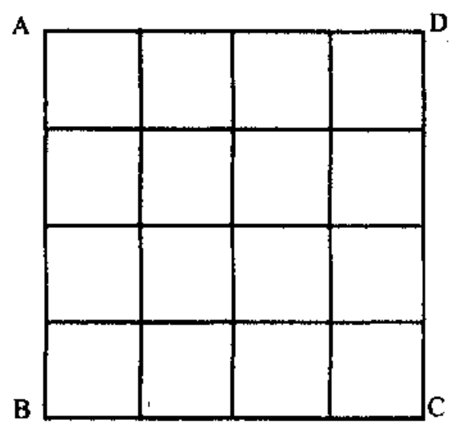


Fig : Q 1. 3 Equidistant parallel and perpendicular lines to side AB.

Q2. Take any three random points P, Q and R on side AB of square ABCD and draw three perpendicular lines through these points to AB. Draw another three lines parallel to AB at distances equal to AP, AQ and AR. Calculate total number of squares formed.

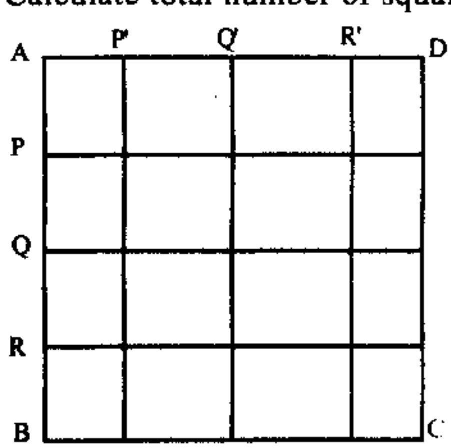


Fig: Q 2. Three parallel and perpendicular lines to AB at random distances.

Chapter - 35
Rosette and School Mathematics

Unlike other origami books, the theme of this entire book is only rosettes. Rosette, a simple circular and disclike model folded from a square paper. An attempt is made in Part II of this book to acquaint the folder with numerous designs and patterns hidden in this `simple' circular model.

After this, yet another attempt is made in Part III to bring out mathematical nuances hidden below an innocent surface of a rosette. The corelations found with NCERT syllabus is a by-product of this attempt; not an intention. This `intention', perhaps, could be served better if more origami models are brought into it. Nevertheless, meanwhile, the mathematics teachers can improvise and add to the basic theme of rosettes to make the related topics activity oriented and fun-filled. Following are some topics from NCERT high school syllabus which are partially/fully related to the theme of rosettes.

Rosettes & School Maths.

(Related Topics from NCERT High-School syllabus)

1. Lines	:	Parallel and perpendicular lines, intercepts, proportional intercepts, division of a line segment into equal parts.
2. Triangles	:	Right angled triangles, congruent triangles
3. Quadrilaterals	:	Squares, rectangles, parallelograms, trapezium etc., some theorems on the same.
4. Circles	:	Radius, diameter, circumference and area of a circle. Chords of a circle, central angle, some angle properties of a circle, arc degree measure of an arc of a circle, length of an arc, inscribed angles into a circle. Sector and sector angle of a circle, area of a sector, area of a segment of a circle. Tangent to a circle, tangents to a circle from a point outside the circle. Some theorems on these topics.
5. Cylinders and Cones	:	Right circular cylinder, cylinder and right circular cone. Volume and surface area of right circular cylinder and right circular cone etc.
6. Trigonometry	:	Sine and tan functions. Drawing sinusoidal waveforms.
7. Miscellaneous	:	Pye charts, linear symmetry of points, lines and triangles. Some 3-D constructions.

3. Demonstration : The DNA helix is now ready for demonstration. The helix looks elegant if you use engineering drawing tracing paper and mark the molecules with different colours.

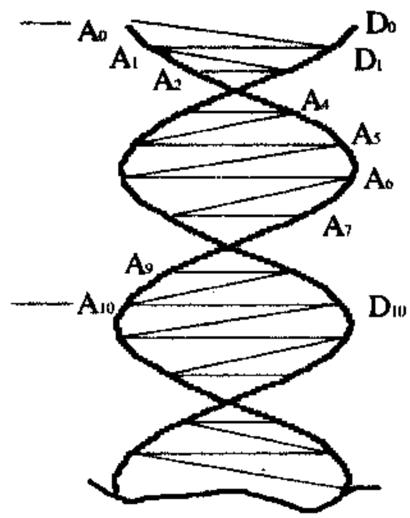


Fig:3. The DNA Helix

Chapter - 34
DNA Helix

The DNA is a crucial molecule of inheritance. It is a code, a memory which controls the function of the species in a regular and predictable manner.

1. Chemical Structure : DNA molecule is structurally a twin helix. Hence it is called the DNA helix. The outside boundaries of the helix are formed by alternate sugar and phosphate molecules. Right and left hand portions of base pairs are held together by hydrogen bonds in horizontal planes. Base pairs are always linked in a definite way. Adenine and thymine are always linked together; so are guanine and cytosine. Only four pairs can exist; A-T, T-A, G-C & C-G.

2(a) & (b) Procedure : Take any thin and tough paper strip. (width preferably about 10 cm and length minimum 30 cm.). Divide the strip into equal rectangles of about 2 cms height. Mark the corners of the rectangles by capital letter S (sugar molecule). Mark letter P (phosphate) between two S molecules. On the centre of the horizontal lines mark H for hydrogen molecules. Pair out T-A & C-G molecules between S and H molecules. After completing all markings, fold and unfold mountain crases.

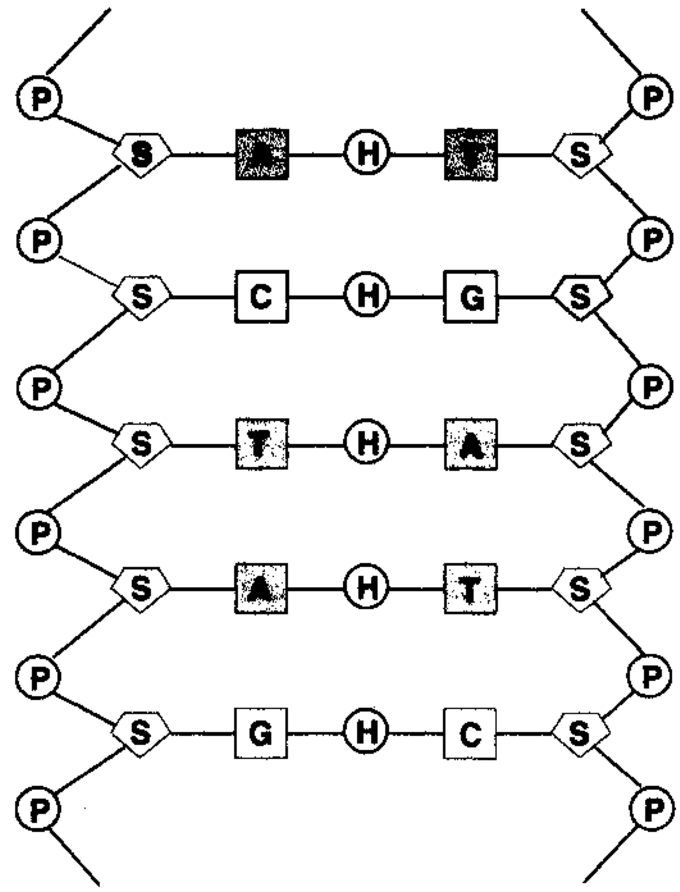


Fig:1. Flattened chemical structure of the DNA Helix

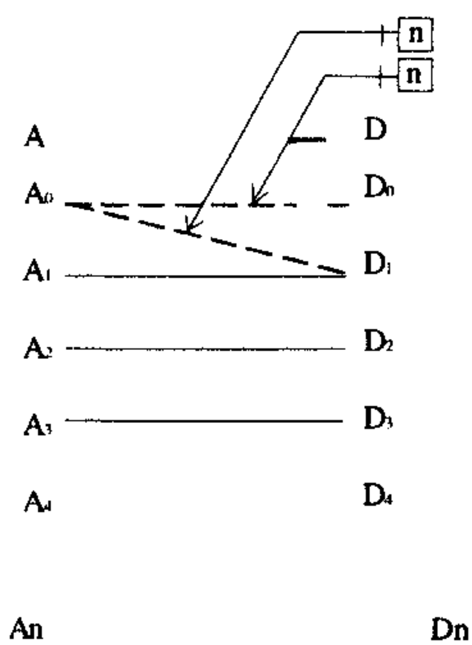


Fig:2 (a). Long Rectangular strip with rosette pleats for DNA model

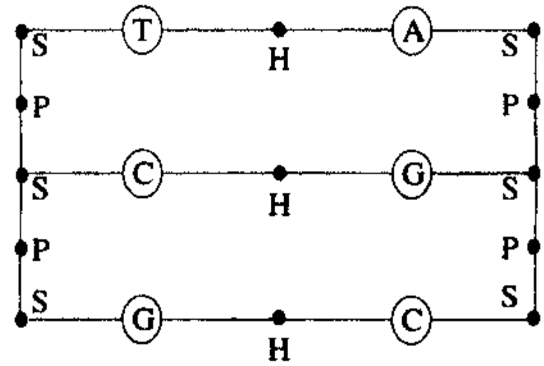


Fig:2. (b) Marking of molecules on paper srtrip

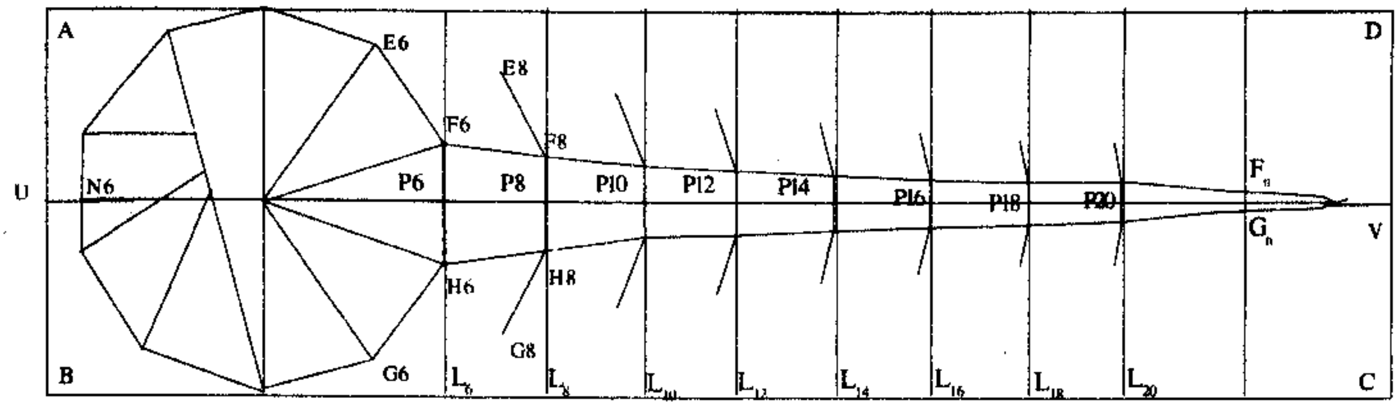


Fig:3. Reduction in contact length with increase in number of division

3. Put rosette with 6 divisions on the paper strip such that corresponding points P_6 overlap exactly. Now adjust line N_6P_6 so that it coincides with the median UV. The line L_6 is now perpendicular to N_6P_6. Take a pencil and mark the corners E_6, F_6, H_6 and G_6 on the strip. Join these points by lines. Repeat this procedure with remaining rosettes.

Join all F points and all G points by smooth curves. You will see that as n increases the segmented arc F_nG_n reduces in length. Finally it converges into a single point. Therefore, as n increases, the line L_n perpendicular to O_nP_n, which is the radius of the circle, intersects the circle exactly in one point. Hence proved.

Fun-Work :

1. What differences do you experience while riding a bicycle with tyre tubes filled with different air pressures?

2. Make a list of experiences from your everyday life, based on 'tangent to a circle' theorem. Do you come across it while 'folding'? (folding with inverted thumb nails?).

Chapter - 33
Tangent to a Circle

Theorem : A line drawn through the end of a radius and perpendicular to it is a tangent TO circle i.e. line drawn touches/intersects the circle in exactly one point.

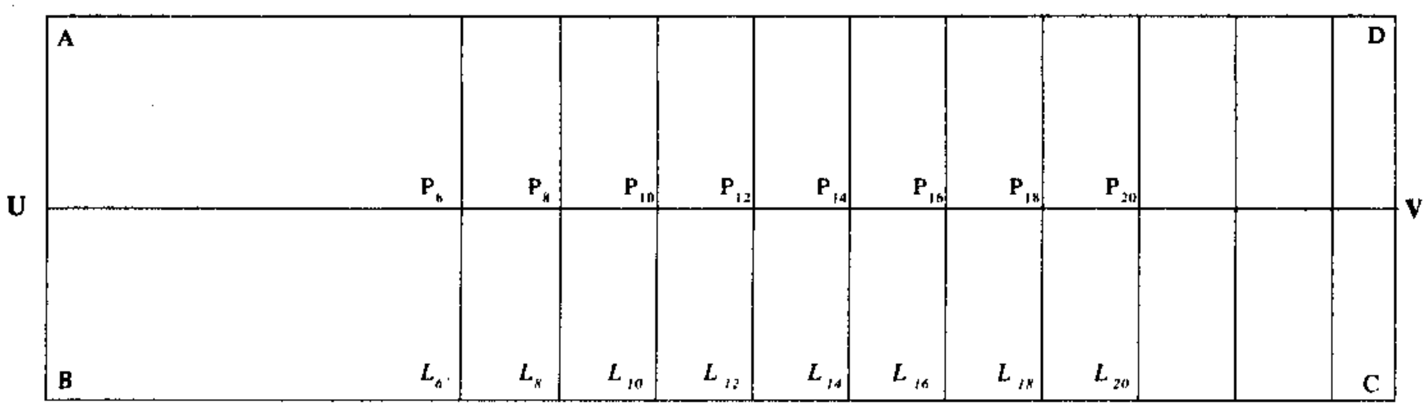


Fig:1. Equidistant division on a rectangular strip

1. Take a suitably long rectangular strip of paper and crease the median UV. On UV mark ten equidistant points as P_6, P_8,...P_n. Crease perpendicular lines L_6,L_8... L_n through the points P_n.(Suffix n corresponds to number of divisions on the rosette).

2. (a) & (b) Take few rosettes with number of divisions n = 6,8,10,12,14,16,18,20 etc. Take a rosette with 6 divisions and mark a mid-point P_6 on its suitable division as shown. Draw a line P_6N_6 through the centre O_6. Repeat this procedure with remaining rosettes.

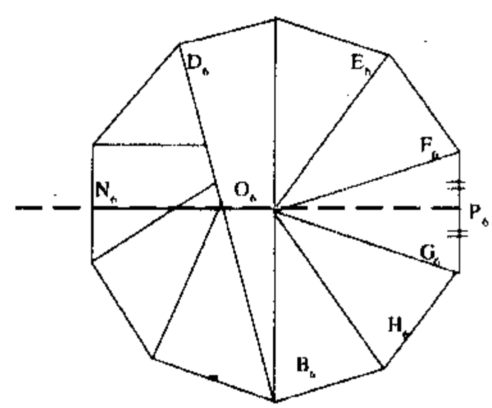


Fig:2. (a) Six division rosette

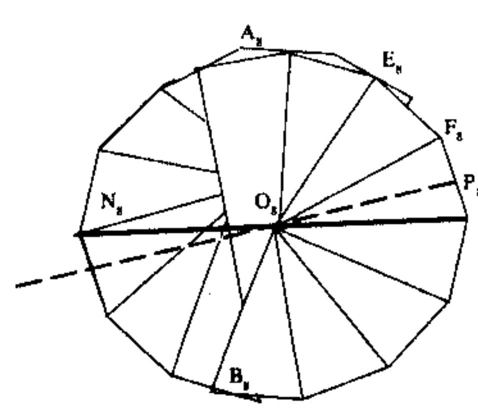


Fig:2. (b) Eight divisions rosettes

Chapter - 32
Perpendicular to a Chord

1. Theorem : The perpendicular from the centre of a circle to a chord bisects the cord.

2. Mark any chord on the rosette by making a crease through any two points on the arc A_1B_1. Let the chord be A_2B_3. (Arc A_1B_1 subtends a central angle of 2δ x 8 = 28^0 x 8 = 224^0 i.e. more than 180^0).

Fold A_2B_3 on itself so that the perpendicular to it passes through O_1. Let the crease be O_1L.

3. To Prove : $B_3L = A_2L$

Proof : $\angle O_1LB_3 = \angle O_1LA_2 = 90^0$ (given)

$O_1B_3 = O_1A_2 = 1/2$ diagonal

O_1L is common to both triangles

$\Delta O_1LB_3 = \Delta O_1LA_2$

$\therefore B_3L = A_2L$

Fun-Work : *Prove the converse of this theorem i.e. the line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.*

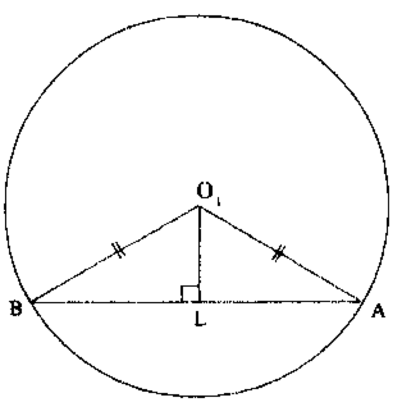


Fig : 1. Perpendicular to chord of a circle

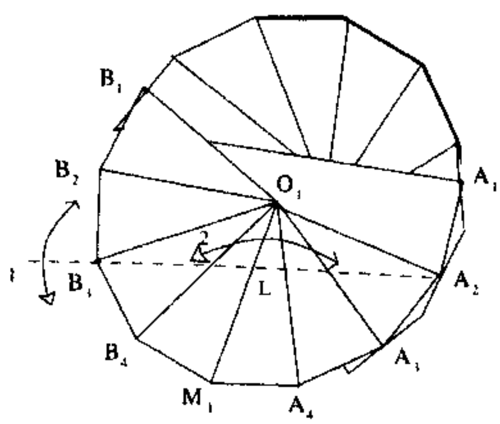


Fig : 2. Chord formed in rosette

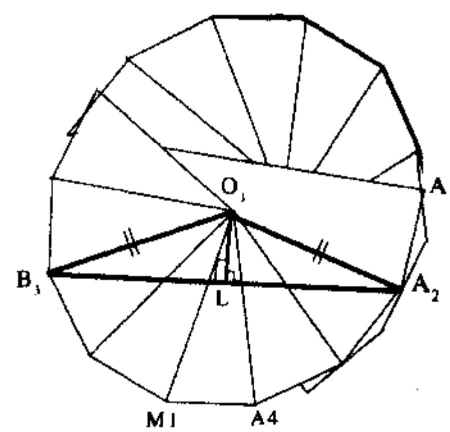


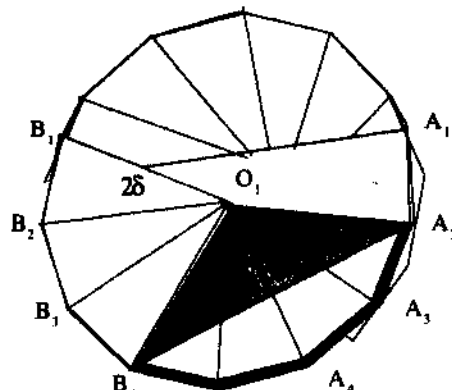
Fig : 3. Perpendicular to chord of a rosette

Theorems on Circles ; Congruent Arcs & Chords

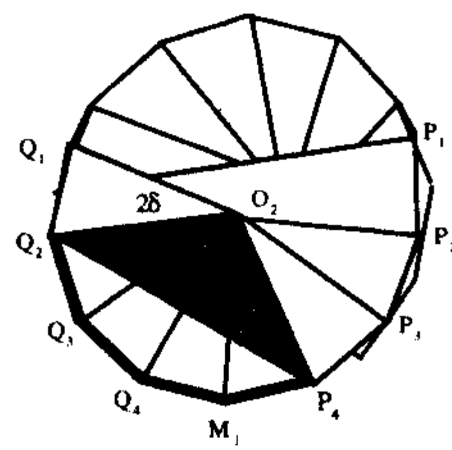
Theorem : If two arcs of a circle (or of congruent circles) are congruent then the corresponding chords are equal.

1. a&b Fold two standard rosettes R_1 and R_2 from two square papers of same dimensions. The two rosettes are congruent. Identify the corners on these rosettes as shown.

Take rosette R_1 and mark with a pencil any arbitrary arc' passing through the corners. Let the arc be B_4A_2. Measure number of divisions included in it (4 divisions). Take rosette R_2 and mark any other arc measuring the same number of divisions i.e.4. Let it be Q_2P_4. Crease the chords B_4A_2 and Q_2P_4 or mark them with a pencil.



(a) Rosette R_1



(b) Rosette R_2

Fig: 1. Two congruent rosettes

To Prove : $B_4A_2=Q_2P_4$

Proof : In Δ s $B_4O_1A_2$ & $Q_2O_2P_4$

$\angle B_4O_1A_2=\angle Q_2O_2P_4= 2\delta \times 4$

$O_1B_4=O_1A_2=O_2Q_2=O_2P_4$

$=1/2$ diagonal

$\Delta B_4O_1A_2=\Delta Q_2O_2P_4$

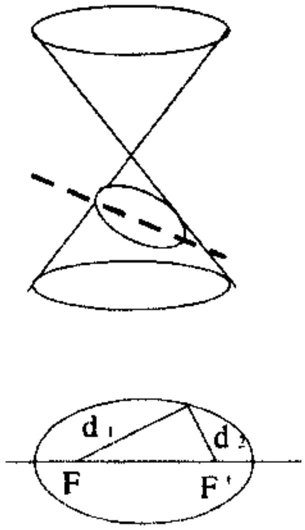
$B_4A_2=Q_2P_4$

You can also overlap chord B_4A_2 directly on Q_2P_4.

Fun Wo k : *Prove the converse of this theorem. If two chords of a circle (or of congruent circles) are equal then their corresponding arcs (minor, major or semicircular) are congruent.*

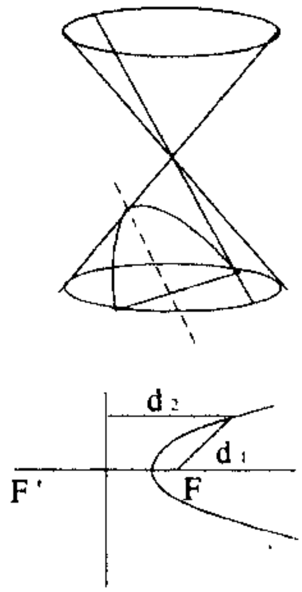
Conic sections

5,6,&7, Formation of conic sections viz., ellipse, parabola and hyperbola.



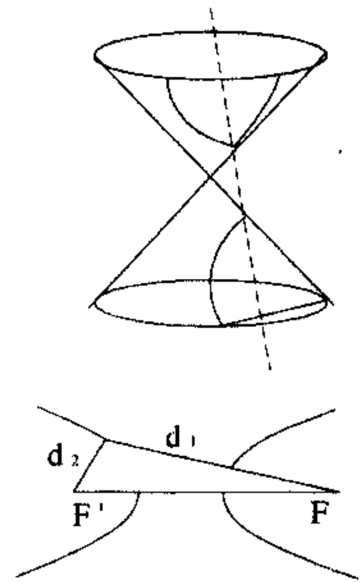
(d_1+d_2 = constant)

Fig:5. Ellipse



($d_1 = d_2$)

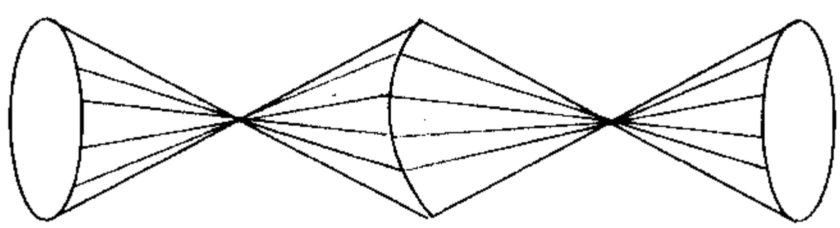
Fig:6. Parabola



($|d_1 - d_2|$= constant)

Fig:7. Hyperbola

***Fun-Work** 1. Make a* model consisting of multiple cones stretched together as shown in the diagram from a single paper.



Chapter - 30
Conic Sections

A single element rosette has two congruent sectors partially overlapped on one another. Therefore two identical cones, joined to each other at apexes, can be obtained from one element. Besides being a handy mathematical model you can utilise it in a variety of applications.

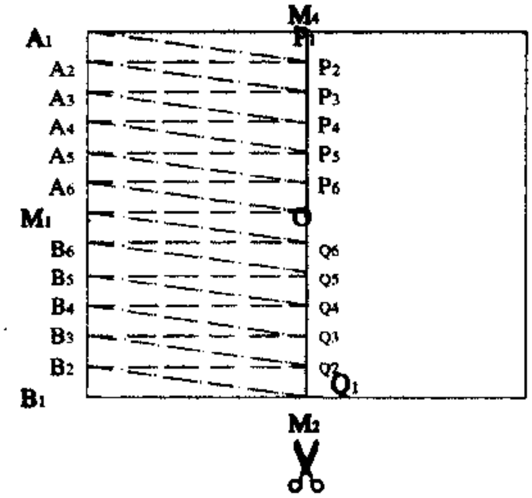


Fig:1. Twelve divisions rosette element

1. Make 12 divisions rosette creases on a thin and tough square paper of about 20x20cm. Use only one element (left hand side element in the figure).

2 (a). Collapse the element into a regular rosette and transpose edges B_1Q_1 and P_1A_1.

2(b). The arc A_1B_1 is clearly in sight. The arc P_1Q_1 is clearly on the backside.

3. Hold the upper layer by the edges as shown and swivel the sides A_1V_1 and B_1V_1 toward one another till the side A_1V_1 aligns with B_3V_1 into the triangular pockets behind A_3V_1. The model has a tendency to open. Hence make it rigid additionally by folding small tip of A_1A_2 over B_2B_3. Repeat below.

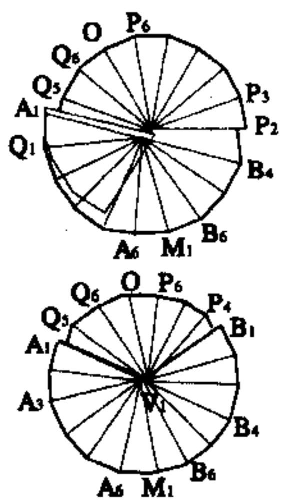


Fig:2. Front and back side of two sectors of one rosette element

4. This model will help you to visualise conic sections namely ellipse, hyperbola and parabola.

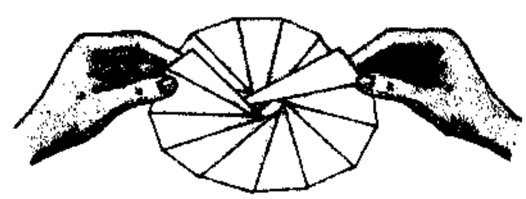
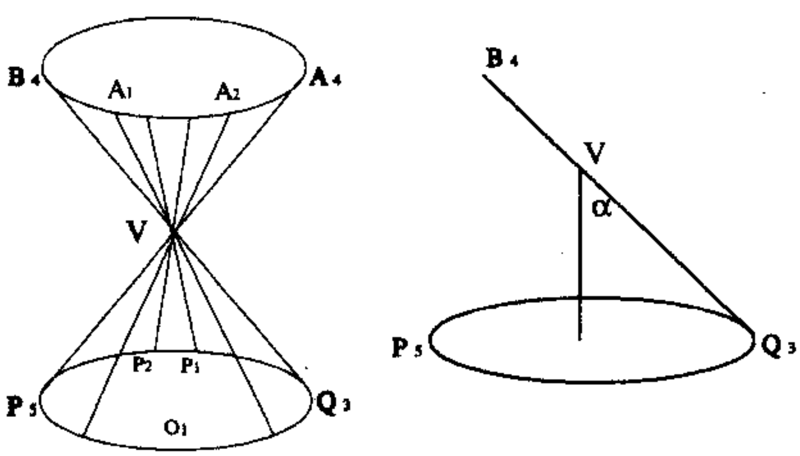


Fig:3. Swivelling two sectors to form a double cone



V, vertex; Q_3V, generator of cone; α, vertex angle

Fig:4. Structure of double cone

4. Open flap RO_1C_1 and swivel the edge C_1O_1 so that it aligns with D_3O_1. Tuck the flap RO_1C_1 into a pocket below O_1D_1 and your cone is ready.

5. Shade the outer surface part of your cone; then fully open it out and lay it flat.

6. Surface area of a cone by ORIGAMI METHOD. The text-book formulae for area of a cone, A, are given in Fig1. But you can calculate it directly from your cone. Let A= area of the cone (curved surface area)

A= area of isosceles triangle $C_1O_1C_2$
= 1/2 b.h ....where:
b = base of triangle $C_1O_1C_2$ = 1/2 unit
h = height of triangle $C_1O_1C_2$ = 1/2 unit
A= 1/2x1/2x1 = 1/4 square units.
A= Ax6........(Six shaded traigles)
= 1/4x6= 1.5 Square units.

7. Here is a sketch of cone made from half rosette element with 16 divisions.

Fun-Work : *Make a close fitting set of a cone and a cylinder to show a relation between their volumes.*

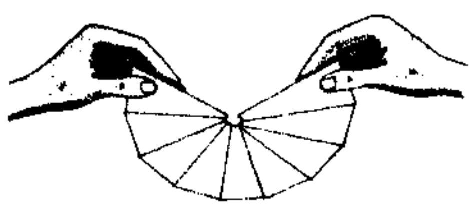


Fig: 4. Swivelling the sector to form a cone

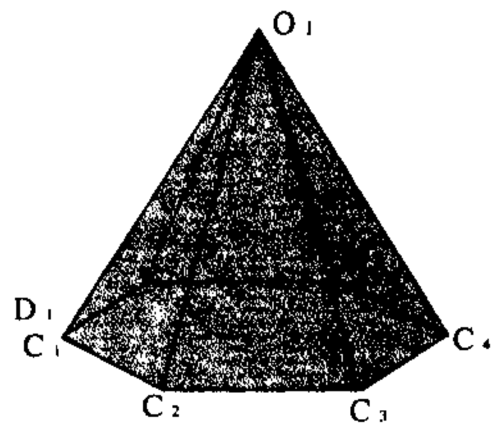


Fig : 5. Cone formed by six segments of a sector

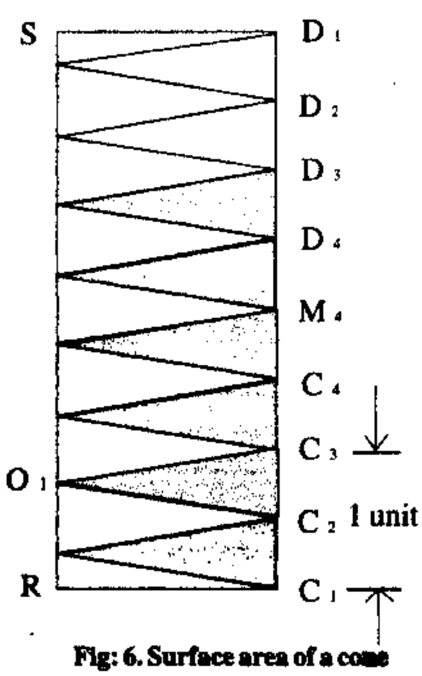


Fig: 6. Surface area of a cone

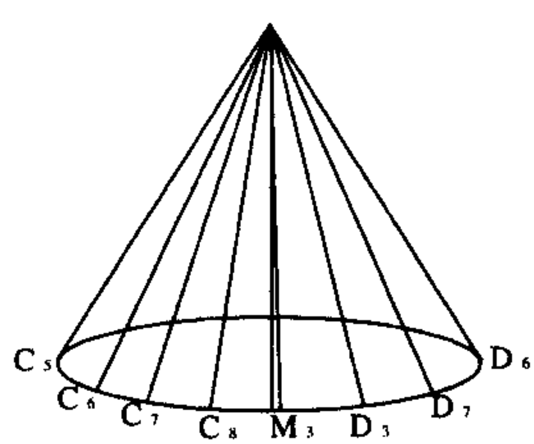


Fig : 7. A cone made from 16 division rosette

Chapter - 29
Right Circular Cone

1. A right circular cone is formed by joining two straight edges of a sector of a circle. If the sector angle θs is reduced then the height **h** increases and the radius **r** of the circular base decreases and vice versa. However the slant height **L** of the cone remains same. If you want a smooth and elegant looking cone then use a rosette element made with more number of divisions. (Fig .7).

2. Take a single element rosette, open it flat and cut it vertically into two equal strips.

3. (a) &(b) . Pleat-fold the half rosette folds on the RHS strip to form a sector $C_1O_1D_1$. Turn the backside of the sector $C_1O_1D_1$ to the front. This surface of the arc will be inside the surface of the cone. (Fig. 3b).

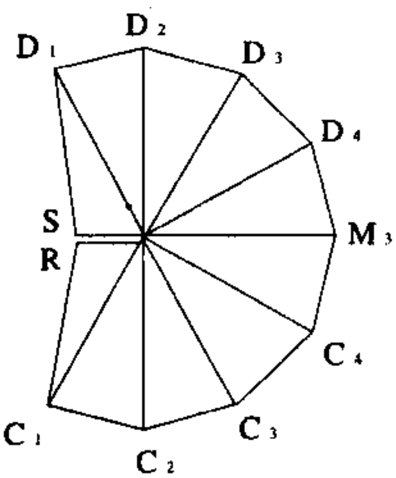


Fig: 3. (a) Sector $C_1O_1D_1$ reversed; Outer surace of the cone

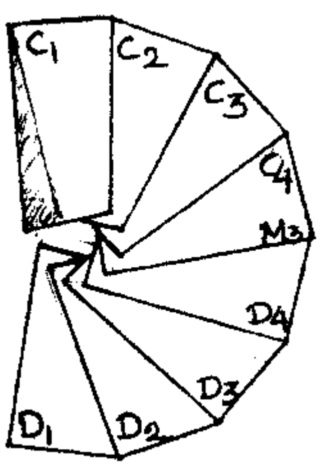
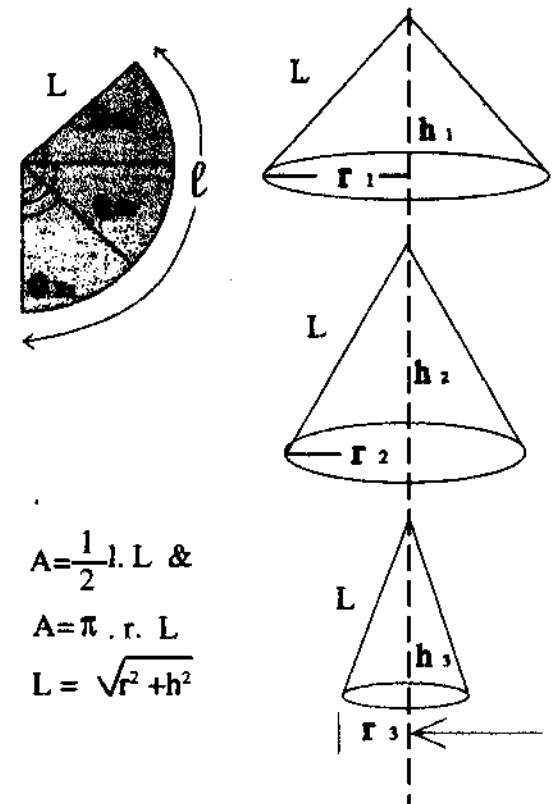


Fig: 3. (b) Sector $C_1O_1D_1$ reversed; Inner surface of the cone



$A=\frac{1}{2} l. L$ &

$A=\pi . r. L$

$L = \sqrt{r^2 + h^2}$

Fig: 1. Cones formed by different sectors of a circle

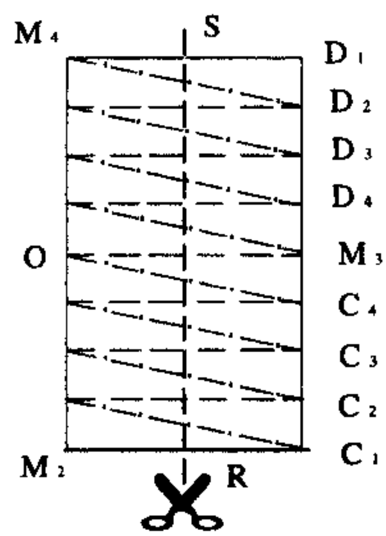
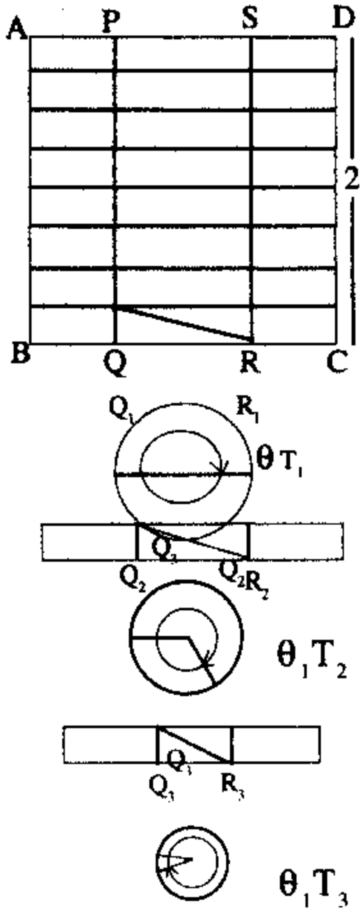


Fig: 2. Rosette element cut vertically into parts



Q_1R_1 max = ?
$2 \cdot \theta_1 \cdot 8 = 180^0$
$\theta_1 = 11.25^0$
$\tan^{-1}(11.25^0) = 0.2$

$$\tan\theta_1 = \frac{1/4}{Q_1R_1} = 0.2$$

$Q_1R_1 = 1.25$ units

Fig: 3. tan θ and limits on maximum width Q_1R_1

Fun-Work : (1) What is the limit on minimum width of cylinder for a given square ? What is the corresponding value of θ ?

Fun-Work : (2) What is the effect of increasing the number of divisions on the volume?

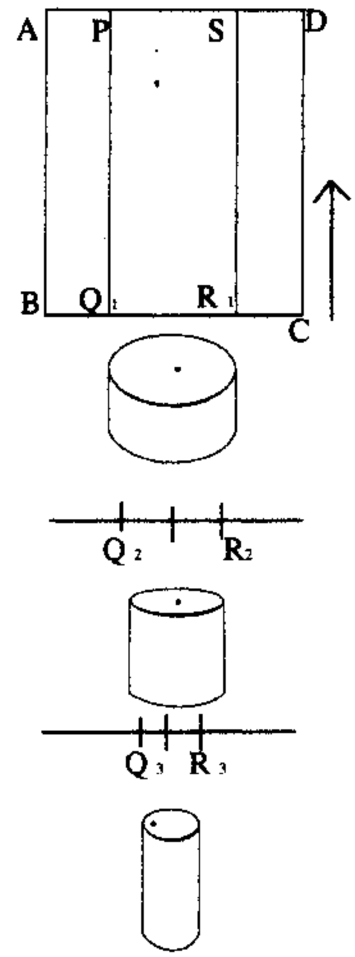
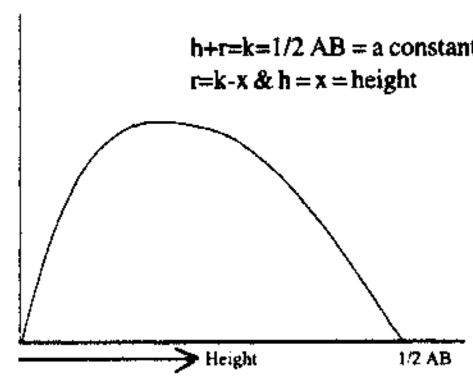


Fig: 4. (a) Variation in volume with increase in height



Volume $V = A.\,h = (\pi r^2).\,h$
$= \pi\,(k-x)^2.\,x$........substituting for r

$$\frac{dv}{dx} = \frac{d}{dx}(\pi k^2x - 2\pi kx^2 + \pi x^3)$$

$= \pi k^2 - 4\pi kx + 3\pi x^2$
$= 3\pi x^2 - 4\pi kx + \pi k^2$........ by rearranging ;
$ax^2 + bx + c$.........this is a quadratic equation;
for V=V max what will be the value of π ?
This is obtained by finding roots of quadratic equation $ax^2+bx+c=O$

Fig: 4(b). Height for maximum volume

Chapter - 28
Right Circular Cylinder : Mathematical Frolic

Here are some clues for you to make use of the cylinders to visualise simple and complex mathematical problems.

1. Circular Base Area ($\overline{A}$)

Base area A of cylinder is essentially the same as the area of the rosette formed by element PQRS.

2. Lateral Surface Area ($\overline{A}_L$)

A_L = Circumference x height. You can shade whole of the outer curved surface of the cylinder. Then unfold the cylinder and lay it flat. Tally observed and calculated areas.

3. tan θ and Limits on Maximum Width

You will notice at once that as the width QR of the cylinder is increased θ is reduced. If $\theta<11.25^0$ the cylinder will not close. You cannot therefore increase the width QR beyond 62.5% of AB or PQ max = 1.25 if AB = 2 units.

Conversely, when you decrease the width QR, θ is increased and there is more overlapping of the layers. Have you noticed this difference while making shallow and deep cylinders?

4. Maximum Volume

Volume V of any shape of cylinder is given by the relation, V=base area x height. Theoretically all shapes, from the deepest (h=1/2 AB) to the most shallow (r=1/2AB) cylinders can be constructed from the same square. Volume at these two extremities will be zero. In between these points, you will get maximum volume V_m. This point of specific height (or radius) could be found out by differentiating the equation for volume and then solving for the roots of quadratic equation. (Neglect the width of interlocking strips).

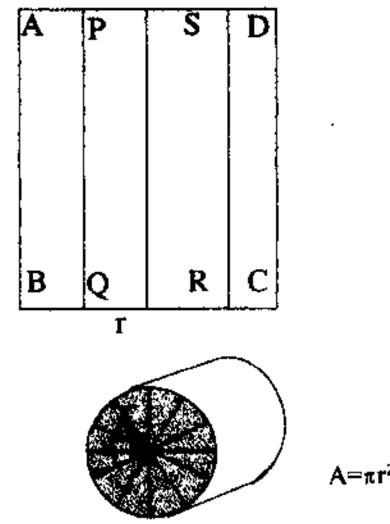


Fig: 1. Area of the circular base

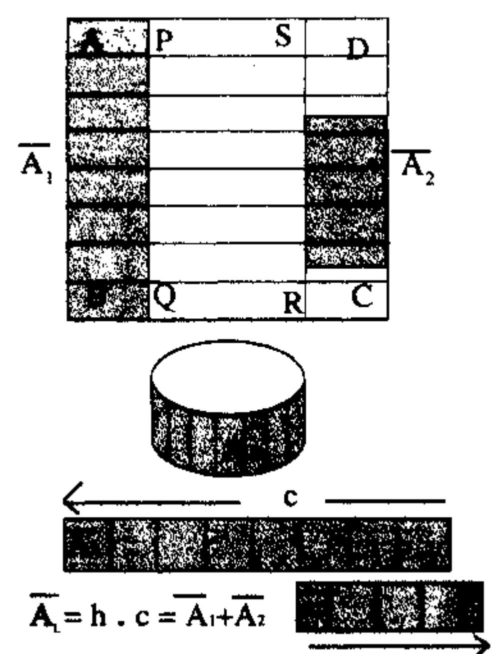


Fig: 2. Lateral surface area

4. Continue to apply force. Did you notice the points M_2 and M_4 getting closer?

5. When the rectangle PQRS forms itself into a rosette stop applying force. The unlocked cylindrical assembly is completed.

6. Carefully transpose the edges R and P of base rosette along with the side walls. This will give more rigidity to the base of the cylinder. To lock the curved walls first fold the strip ABUT and then strip WVCD, over one another. This will hold cylindrical structure firmly in place.

7. Cylinder is ready

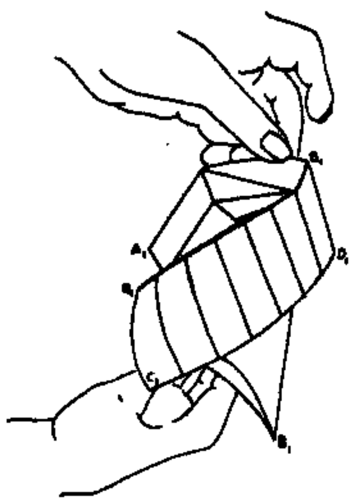


Fig: 4. Application of rotational forces continued

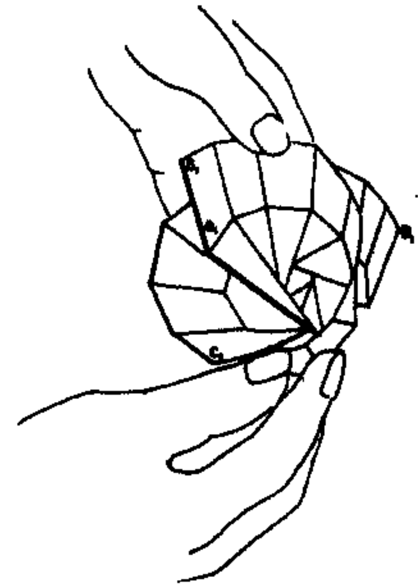


Fig: 5. Formation of base rosette

Fig: 6. Transposition of rosette edges

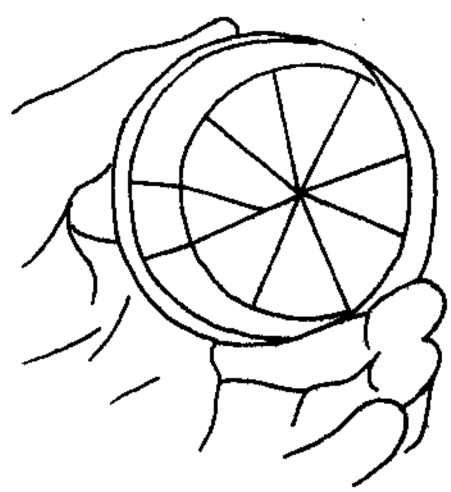


Fig: 7. Interlocked edges : cylinder with a firm assembly

Chapter - 27
Right Circular Cylinder : General Techniques

Here are some techniques to master the art of making cylinders.

Precautions

(a) Use thin and tough paper, the one used in engineering drawing is ideal. **(b)** Do not make unwanted creases. The median M_2M_4 should never be creased though it is necessary to marke points M_2 and M_4. **(c)** The creases should be sharp and accurate. You can use non-cuting narrow edges like plastic knife, blunt screwdriver edge etc. However, these creases should be scored again with inverted thumb nail in the usual way. (d) The square will turn itself gradually into a twisted form as shown in the fig:2 if all creases are correct and straightened. Check all the creases before you proceed.

Prodecure :

1. The four different arrows in Fig. 1 show the directions of forces which you have to apply gradually and simultaneously. It is interesting to do a dress rehersal with empty hands before you start with the square.

2. Hold the pre-creased square by the points M_2 and M_4 in your two hands with a flexible grip.

3. Apply gradual rotational forces to M_2 and M_4 in opposite directions and simultaneously push M_2 and M_4 towards each other. You should first evoke the creases during rotational movements and then follow them with a great alacrity. Straighten the creases if necessary.

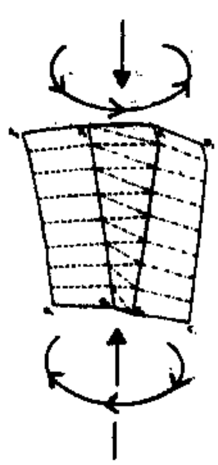


Fig: 1. Directions of 4 forces to be applied simultaneously

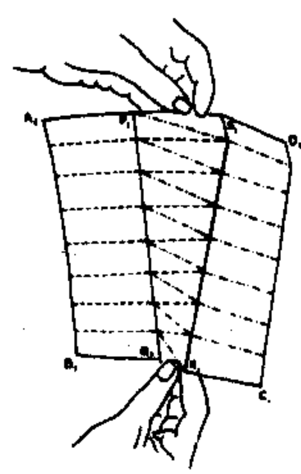


Fig:2. Holding the square correctly

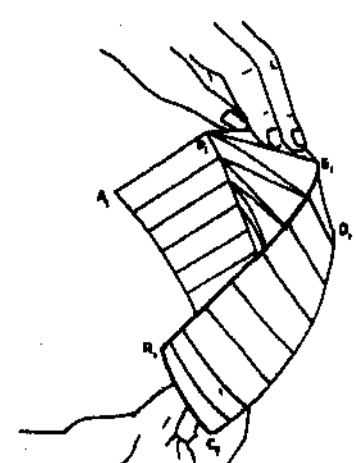
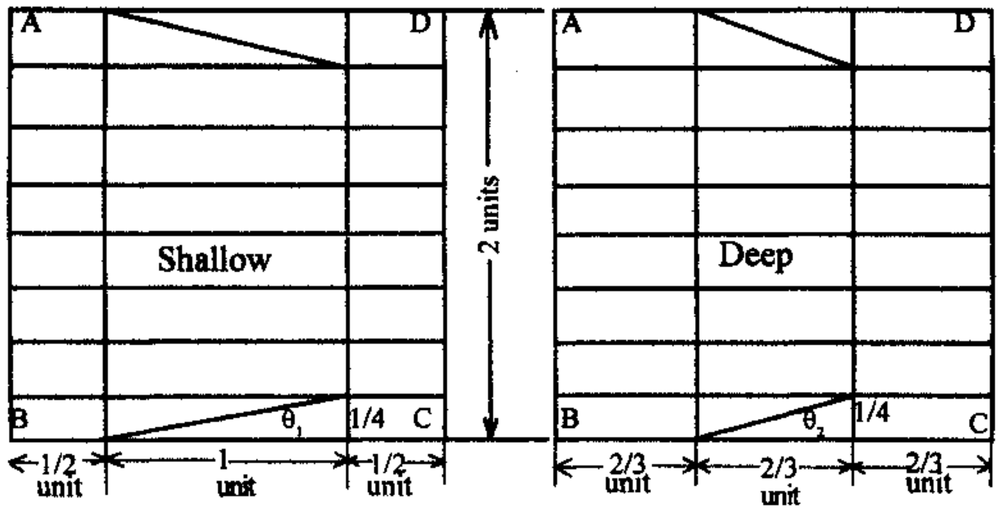


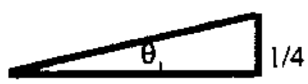
Fig: 3. Gradual application of all four forces

Chapter - 26
Comparision of Mathematical Properties for Shallow and Deep Cylinders

1. Proportions of widths



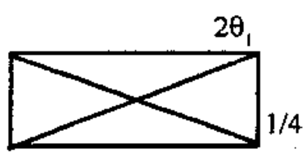
2. Angle made by one division



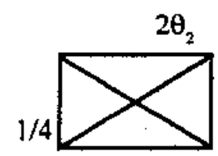
$\theta_1 = \tan^{-1}(1/4)$
$= 13.8^0$
$= 14^0$

$\theta_1 = \tan^{-1}\left(\frac{1/4}{2/3}\right)$
$= \tan^{-1}(3/8)$
$= 20.5^0$

3. Sector angle made by one division

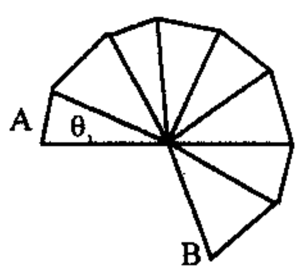


$2\theta_1 = 28^0$

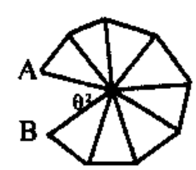


$2\theta_2 = 41^0$

4. Sector angle made by one side of the square(should be greater than 180 ^{0})

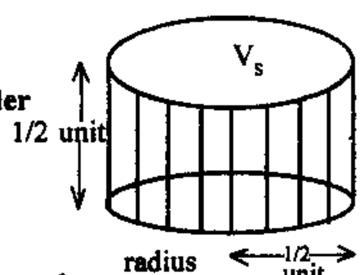


$2\theta_1 \times 8 = 224^0$

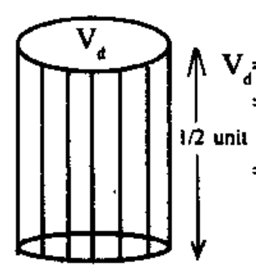


$2\theta_2 \times 8 = 328^0$

5. Volume of cylinder V=Base area X height= $\pi r^2 x h$



$V_s = \pi (1/2)^2 \times 1/2$
$= 0.39$ Cubic Units
$= 0.4$ Cubic Units



$V_d = \pi (1/6)^2 \times 2/3$
$= 0.2326$ Cubic Units
$= 0.2$ Cubic Units
almost half of V_s

6. Extra overlapping angle (should be less than 180^0)

$\frac{448^0 - 360^0}{2} = 44^0$

$\frac{656^0 - 360^0}{2} = 148^0$

7. Total sector angle made by two layers(should be greater than 360^{0}and less than 720^0)

$224^0 \times 8 = 448^0$

$328^0 \times 8 = 656^0$

Chapter - 25
Right Circular Cylinder (Deep)

The procedure for making deeper cylinders is same as for shallow cylinders except for the dimensions of the rosette element PQRS.

1. Divide the square into 3 equal columns. Let PQ and SR be valley and mountain creases respectively. Then divide the square into 8 equal horizontal rows. Make valley creases to the left and mountain creases to the right hand side of SR. You can also make 10 or 12 rows for better results.

2. Mountain fold the diagonals in the rosette element PQRS. Fold and unfold the narrow strips TU and WV. Repeat the creases, if necessary.

3. Start collapsing the precreased square gradually through the stages as shown in the chapter 27 on General Techniques for Cylinders. Your cylinder is ready. Use more number of divisions to make it more elegant.

Did you notice excessive overlapping of layers in deep cylinders compared to shallow ones? Mathematical explanation is given in the next chapter.

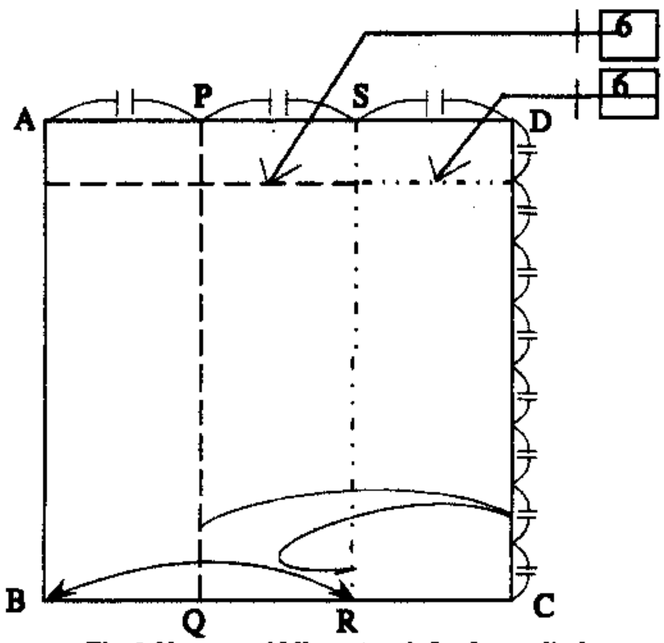


Fig: 1. Narrow middle rectangle for deep cylinder

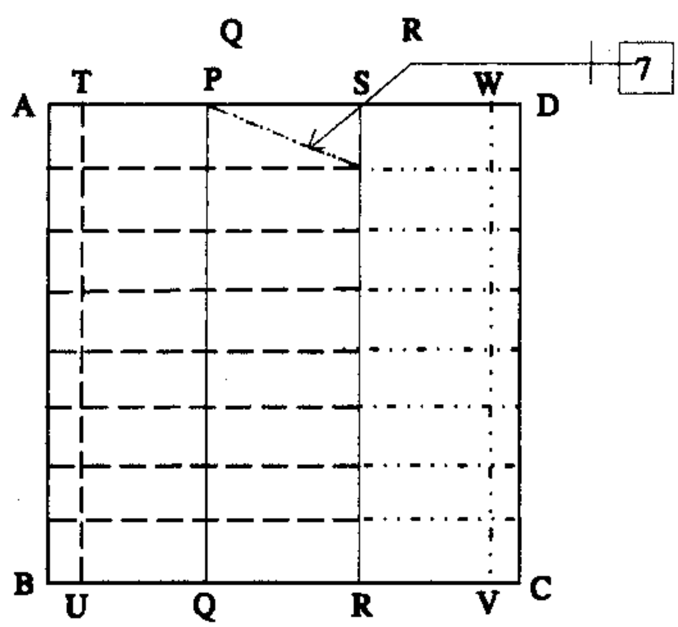


Fig: 2. Regular eight division folds

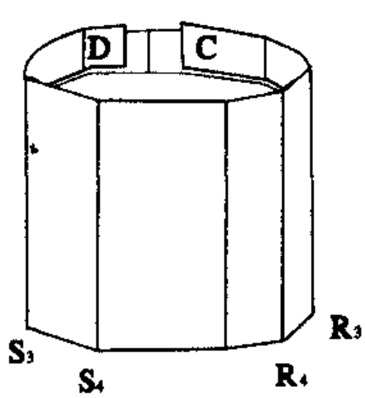


Fig: 3. Deep cylinder with base and side locking completed

Chapter - 24
Right Circular Cylinder (Shallow)

Next time think of packing the gifts for your dear ones in close fitting cylinders with lids. A wide range of shapes is possible from the same size of square papers.

1. Mark M_2, M_4. Do not crease the median M_2M_4. Mark P,Q,R and S as shown. Valley fold and unfold PQ and then mountain fold and unfold SR.

2. Divide the square into 8 equal horizontal divisions. Divisions to the left hand side of SR are valley folds and those to the right hand side are montain folds.

3. Carefully mountain fold and unfold the diagonals in the rectangular element PQRS (standard rosette folds).

Take AT approximately=1/5 AP. Valley fold and unfold TU. Take WV slightly smaller than AT. Mountain fold and unfold WV. These strips will help in the locking the edges of cylindrical assembly.

4. Carefully collapse the precreased sqaure into cylindrical assembly as shown in chapter no. 27 on General Techniques for Cylinders. Interchange the positions of edges R and P. Lap the partially overlapping bases perfectly and interlock the strips TU and WV by folding over one another.

Your cylinder is ready for delivery !

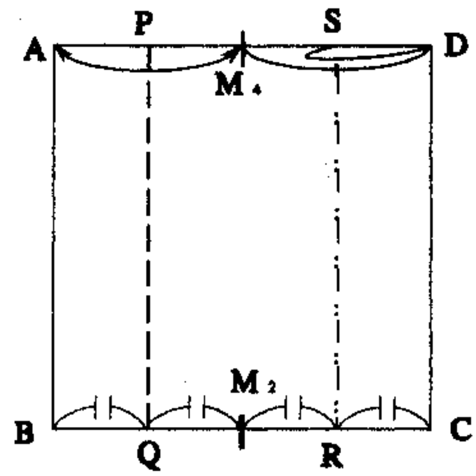


Fig: 1. Wider middle rectangle PQRS for shallow cylinder

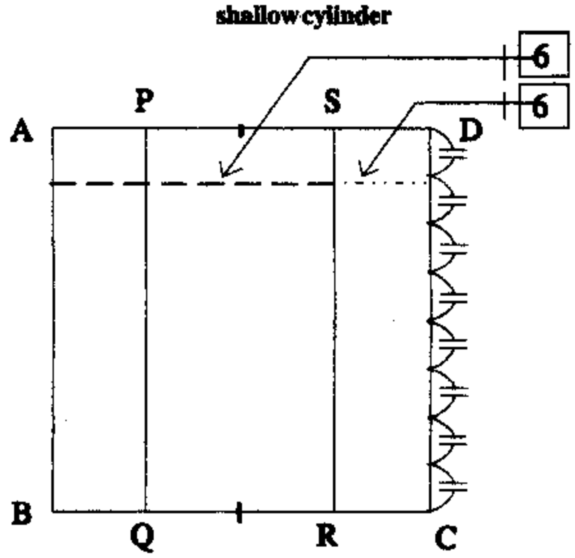


Fig: 2. Regular eight division folds

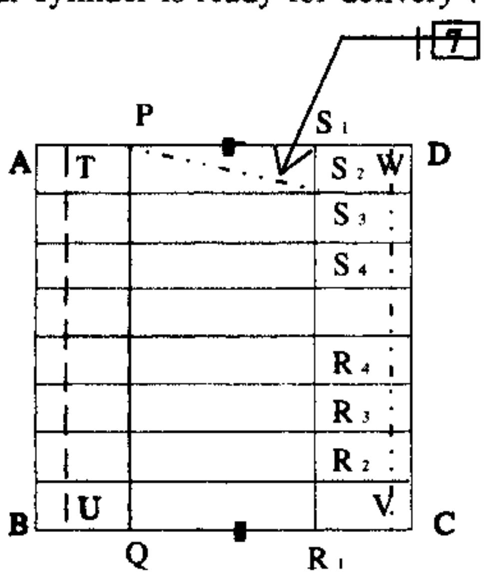


Fig: 3. Diagonal mountain folds for circular base

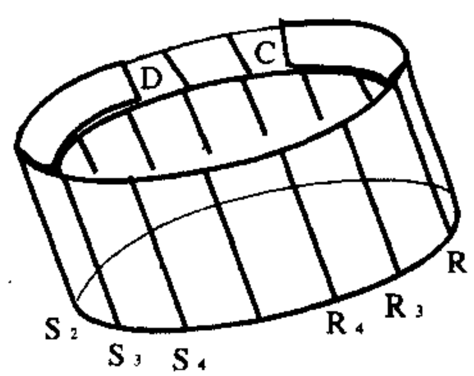


Fig: 4. Cylinder with base completed

Fun-Work : (1) Make a shallow cylinder with a separate lid.

Area of Circle

= Area of sector AO_1B + area of sector $B'O_1A'$ = 2+1.2 = 3.2 sq. units

5. Area of one isosceles triangle

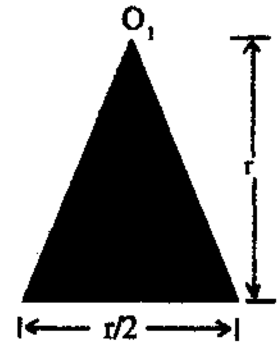
= 1/2 base x height

= 1/2 x r/2 x r

= 1/2 x 1/2 x1

= 1/4 square units.

(r=1 unit)



Δ Area = ¼ units.

Fig :5. Area of one isosceles triangle for 8 divisions rosette

6. Shown here is a table of areas of rosettes with different number of divisions.

n = number of divisions in the rosettes

$\overline{\Delta}$ = area of one triangle in sq. units

AO_1B = number of triangles in sector AO_1B

$B'O_1A'$= number of triangles in sector $A'O_1B'$

A = area of circle in square units

n	$\overline{\Delta}$	AO_1B	$B'O_1A'$	$A=(AO_1B+B'O_1A')X\overline{\Delta}$
8	1/4	8	4.83	3.2
10	1/5	10	5.8	3.16
12	1/6	12	6.9	3.15
14	1/7	14	8	3.14

Fig :6. Table of areas for different rosettes

Area of Circle

Rosettes can also be used to derive the formula for the area of a circle. Accurate results are obtained from a rosette made with more number of divisions.

1&2. Take a regular 8 divisions rosette and mark points O_1,A,B,A'*and* B' as you did for deriving the circumference. Additionally draw lines AO_1 and BO_1 on the front side and lines $A'O_1$ and $B'O_1$ on the back side. With a pencil, shade the sector AO_1B on the front side and the sector $B'O_1A'$ on the back side.

3. The total shaded area forms the area of the circle.

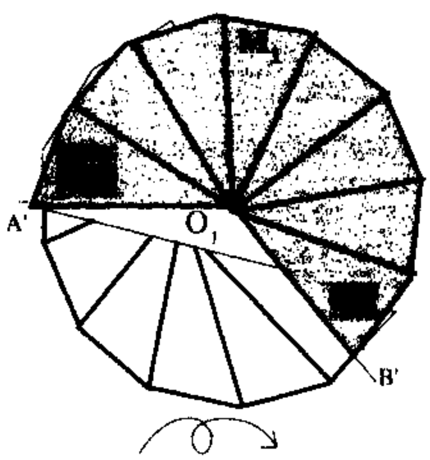


Fig :1. Area of Shaded Sector O_1AB

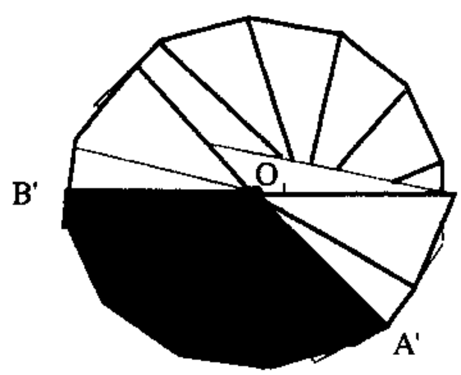


Fig :2. Area of Shaded Sector $O_1A'B'$

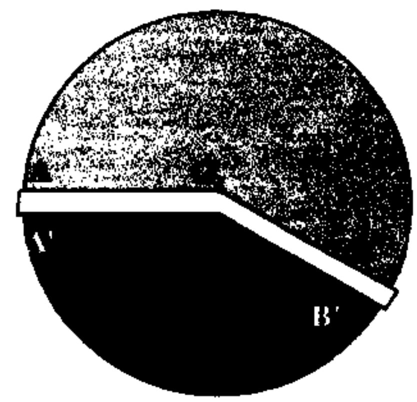


Fig :1. Total area of sector O_1AB and $O_1A'B'$

Assume side of square = 4 units in all cases.

4. Now open out the rosette to form a flat rectangle (r=1 unit).

Area of sector AO_1B

= sum of congruent triangles formed on RHS

= 8 x area of one Δ = 8x1/4

= 2 sq units

Similarly,

Area of sector $B'O_1A'$

= sum of trinagles formed on LHS

= (1/2+4+1/3) x 1/4

= 1.2 sq. units

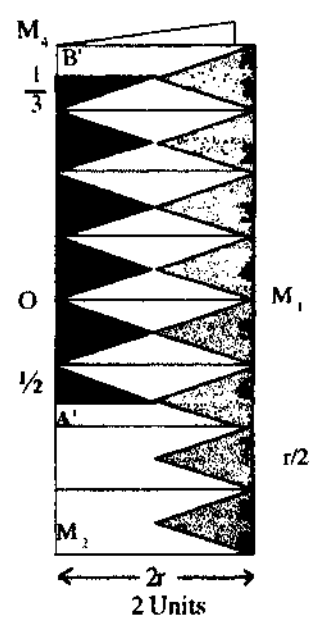


Fig :4. Area of circle converted into congruent isosceles triangles

Chapter - 22
Circumference of a circle

Rosettes can be used to derive the formula for the circumference of a circle. A rosette tends to be a perfect circle as we increase its number of divisions and more accurate results are obtained.

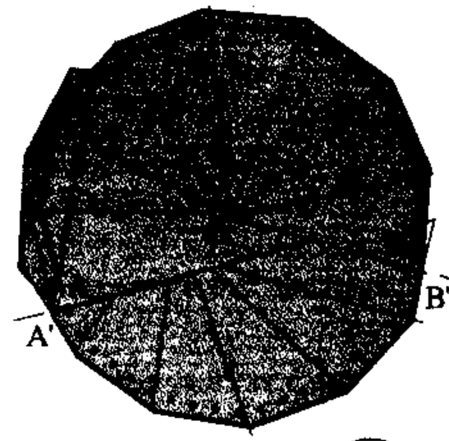


Fig :1. Circumference = $l\widehat{AB} + l\,B'A'$

1. Take a regular 8 division rosette. Mark the centre O_1, and the two corner points A and B. Mark points A' and B' corresponding to A and B, but on the back side. With a pencil draw a continuous line in a clockwise direction along the rim from A to B on the front side and from B' to A' (anticlockwise) on the back side. The total length marked with pencil forms the circumference of the circle. Read $l\,\widehat{AB}$ as `length of arc AB'.

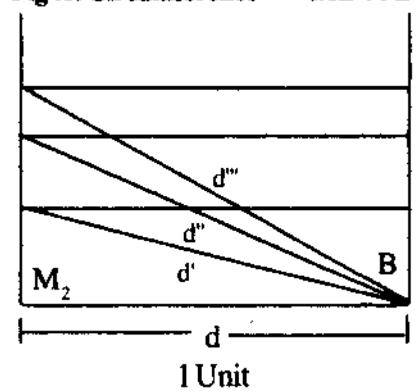


Fig :2. Change in diameter with number of divisions

2. In our rosette method of deriving the value of the circumference of a circle we have assumed that when the number of divisions made in a rosette are increased the diagonal tends to be equal to the width of the rectangle i.e. 1 unit, giving more accurate results. It will be interesting to compare the origami method results with mathematical method in which diameter of the circle is taken as 1 unit.

3. Now open out of the rosette to form a flat rectangle.

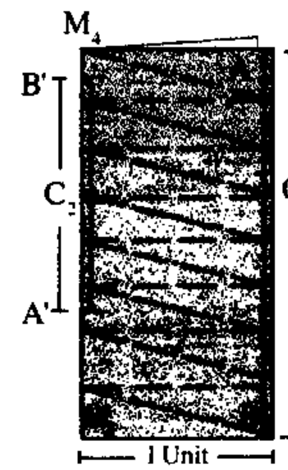


Fig :3. Circumference $C = C_1 + C_2 / n/2$

4. Here is a table of circumferences of a circle with increasing number of divisions. where;

n = number of divisions of the rosette rectangle

C_1 = length of arc AB in terms of the number of divisions

C_2 = length of arc $A'B'$ in terms of the number of divisions

$C_1 + C_2$ = circumference of circle in number of divisions

$C = \dfrac{C_1 + C_2}{n/2} =$ circumference of circle in units

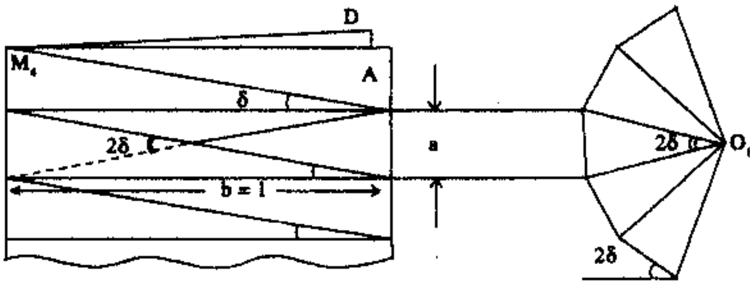
Make rosettes of 10,12 & 14 divisions and tabulate your results in a similar way.

This method, conversely, can be used to find the value of π (pi).

n	C_1	C_2	$C = \frac{C_1 + C_2}{n/2}$
8	8	4.83	3.2
10	10	5.8	3.16
12	12	6.9	3.15
14	14	8	$\left(\frac{22}{7}\right) = 3.14$

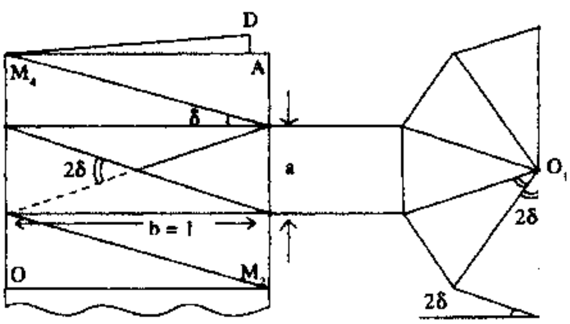
Fig :4. Table of circumferences for increasing number of division

Fold four rosettes with n = 2,6,8 and 10 from square papers of same size. Tally your results with the diagrams given below.



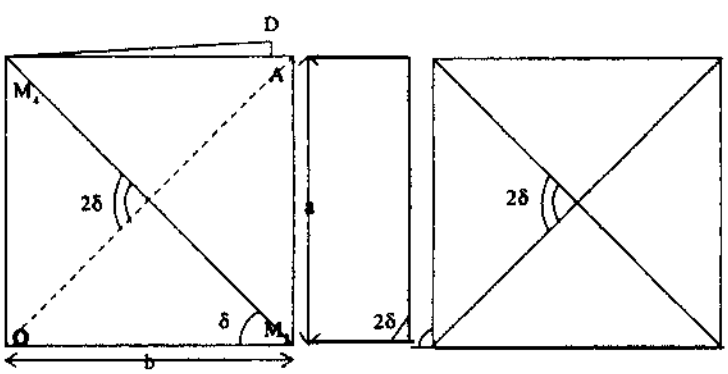
n = 8
tan δ = ¼
δ = 14°

Fig:4. Eight division rosette



n = 6
tanδ = 1/3
δ = 18°

Fig:5. Six division rosette



n = 2
tanδ = 1
δ = 45°

Fig:6. Two division rosette

Fun Work : *(1) Make rosettes with proportion of a/b =2 and 4. Take enough long strip of paper to accommodate atleast 4 divisions.*

(2) How will you make rosettes with 2δ = 20^0 and 30^0 ?

Chapter - 21
Rosettes and tanδ

1. tan δ Curves : Unlike sine and consine curves the tan curve is neither smooth nor continuous. The ratio tan δ is somewhat linear and gradual in variation up to 45^0. After 45^0 it goes on increasing rapidly. Soon it reaches infinity and becomes discontinuous. Hence it cannot be used for representing continuously changing parameters like A/C voltage, sound wave etc. But it is very useful in measurement of sides and angles in engineering and other structures. In electrical measurements small loss angle is denoted by tan d. Active and reactive components of electrical power are shown in Fig 1 by vectors **b** and **a** respectively.

2. Rosette Gauze: If you want to combine contours of different rosettes in one diagram then it should look as in Fig. 2. It is rewarding to make such a rosette gauze.

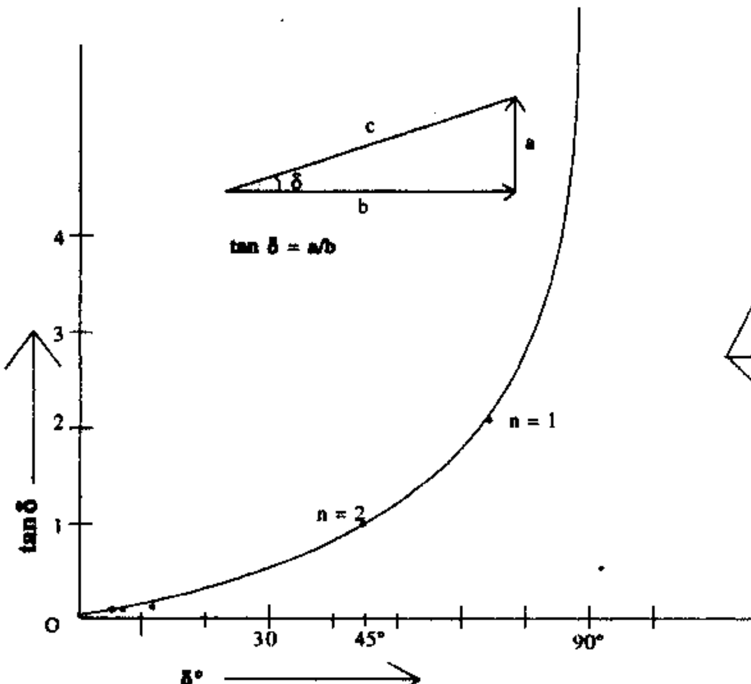


Fig:1. Variation of tan δ with δ

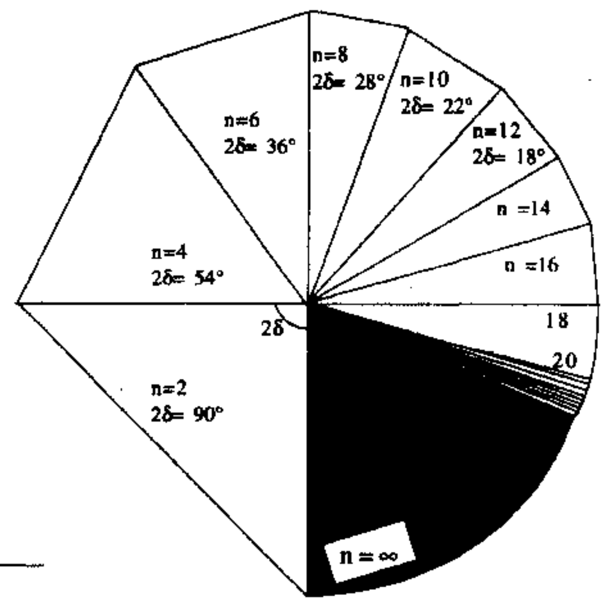


Fig : 2. Rosette Gauze

3to6. Rosette with different number of divisions are aesthetic expressions of the mathematical relation tan δ. When the number of divisions is varied the ratio tanδ a/b also varies. This variation shows up, first, up to 45^0, in the "roundness" of the rosettes, and then, after 45^0, in the out of proportion and clumsy shape of rosettes.

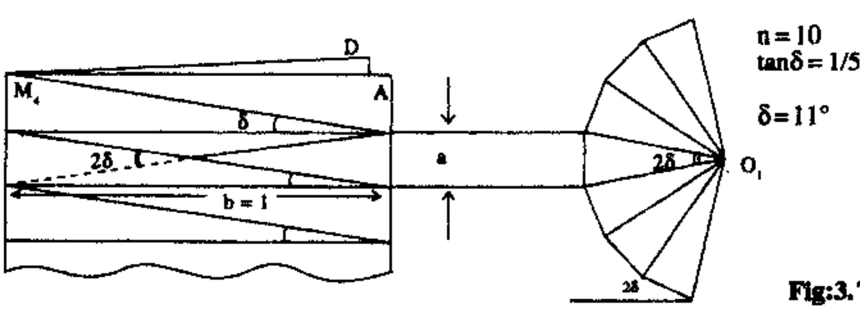


Fig:3. Ten division rosette

Chapter - 20
Rosette with Continuous 360^0 Arc

1(a)&(b) Many a times it is necessary to have a rosette with a full continuous arc which spans 360^0. In a standard rosette the arc AB is not continuous after point B. To span the full circle this arc must contain at least 13 divisions at a stretch. Increasing merely the number of divisions in the given rosette element does not increase the length of the arc AB.

Let us find a simple solution to this problem.

2. Take a standard rosette and open it flat to the original square. Cut it along the median M_2M_4 to form two single rosette elements. Turn behind the right element. Have you noticed reversal in valley and mountain folds?

3. Overlap and align exactly one bottom division of the turned rosette (right part) behind the top division of the left part. This will form a creased rectangle of dimensions slightly less than 4:1.

4. Pleat fold along the creases to form a rosette, with a continuous arc which spans an angle of 420^0 (28^0x15=420^0). Gluing of the two sections is not necessary. The rosette has two layers of a full arc :

Fun-Work: *How will you construct a single layer rosette with a full arc?*

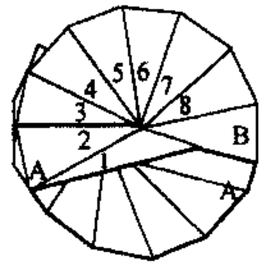


Fig:1. (a) Rosette with angular measure of Arc AB= 224^0

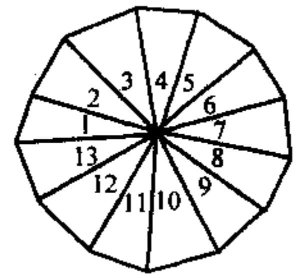


Fig:1. (b) Rosette with angular measure of Arc = 360^0

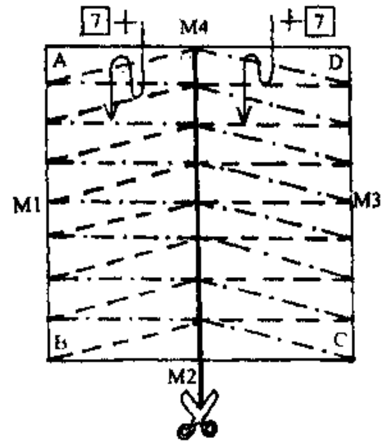


Fig:2. (a) Standard rosette opened and cut into two parts

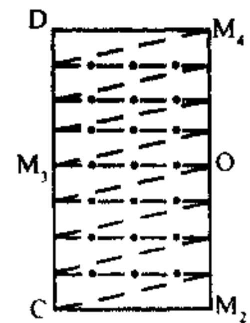


Fig:1. (b) Right part turned around for alignment

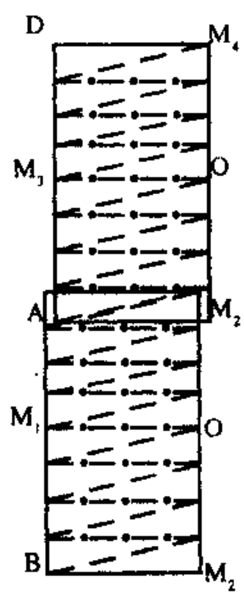


Fig:3. Aligning two parts of one rosette

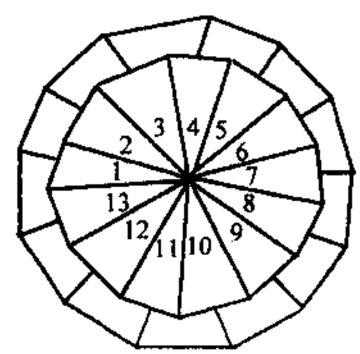


Fig: 4 Full arc rosette with double layers

Chapter - 19
Circle, Rosette and Regular Polygon - A Comparison

1. Substitute for Circle : A rosette merges into a circle when the number of divisions in its element is increased. Hence a rosette is a practical substitute for a circle.

2. Graticule : A rosette has a graticule of equiangular radii. This feature helps in identification and study of commonly used parameters of circle like chord, arc, sector, central angle etc. It also helps in proving many theorems on circle.

3. Rosette and Regular Polygon : A reosette is a closed polygon like structure. The polygonal structure can be completed through 360^0 by increasing the length of the rosette element, if necessary. Theoretically, all regular polygons can be constructed from a rosette.

4. Reversibility : A rosette can be stretched flat again into the original element i.e., rectangle. This feature helps in simplification and measurement of properties of circle like circumference, area etc.

Beyond Circles & Polygons : A rosette can be readily modified into more complex structures like right circular cylinders, cones, helixes, spirals, etc.

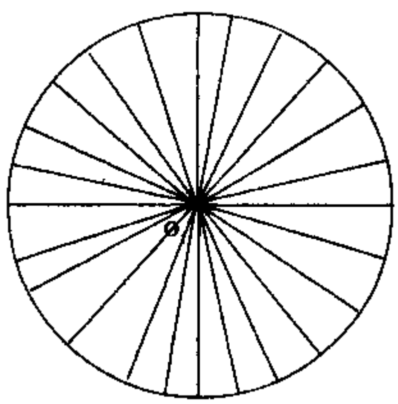


Fig:1. Circle. A rosette with a large number of divisions

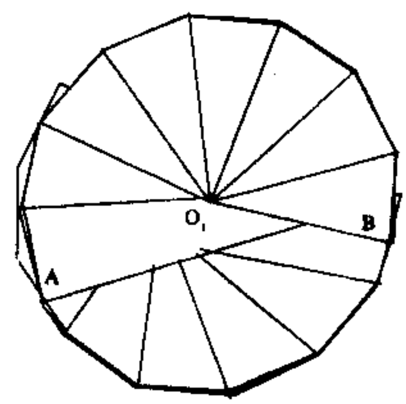


Fig:2. Rosette with graticule

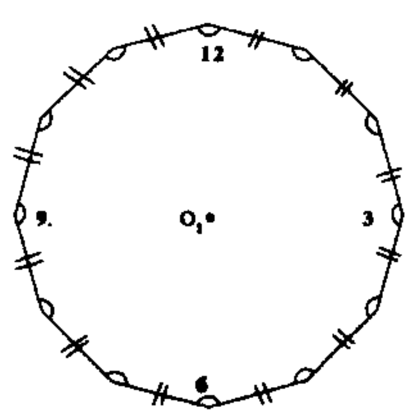
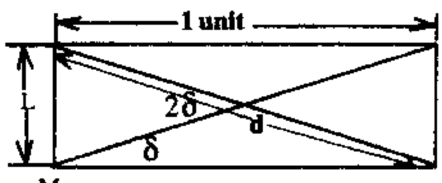


Fig:3. Regular Polygon

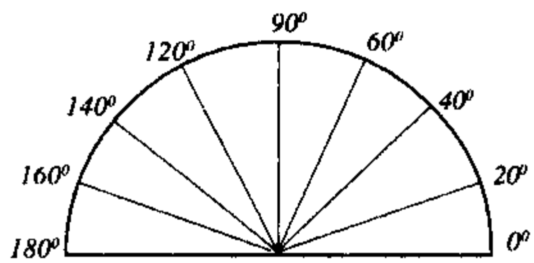
Table of Mathematical Properties (Cont...)

n	l=2/n	d	δ°	2 δ°	N
11	0.18	1.016	10.3	20.6	17.5
12	0.167	1.014	9.5	19	18.9
13*	0.15	1.011	8.8	17.6	20.5
14	0.14	1.010	8.1	16.2	22.2
15*	0.13	1.009	7.6	15.2	23.7
16	0.125	1.008	7.1	14.2	25.4
17	0.12	1.007	6.7	13.4	26.9
18	0.11	1.006	6.3	12.6	28.6
20	0.1	1.005	5.7	11.4	31.6
24	0.08	1.003	4.7	9.4	38.3

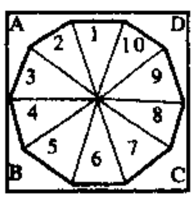
***Method for making n=9,13,15,17 divisions are not shown in this book**



Measurement of diagonal



Fun-Work* : *(1). Construct an origami protractor with a least count of 10°.



Fun-Work* : *(2) Construct a decagon from a given square

Chapter - 18
Table of Mathematical Properties

In an earlier chapter, you have seen how each property of a standard rosette with 8 divisions was found out. The same method is applied here for rosettes made with different divisions. The results are tabulated for ready reference.

You are acquainted with :

n = total number of divisions made in a rosette

l = length of one division in units
= 2/n (side of square =2 units)

d = length of a diagonal in units
= $\sqrt{1^2+l^2}$

δ = angle made by a diagonal with a horizontal side, in degrees
= $\tan^{-1}(l)$

2δ = angle subtented by each division of a rosette at the centre in degrees.

N = number of sides of a regular polygon made by a rosette = $360^0/2\delta$

n	l=2/n	d	δ^0	$2\delta^0$	N
1	2	2.24	63.4	127	2.8
2	1	1.41	45	90	4
3	0.66	1.19	33.7	67.4	5.3
4	0.50	1.12	27	54	6.7
5	0.40	1.08	22	44	8.2
6	0.33	1.05	18	36	10
7	0.29	1.04	15.9	31.8	11.3
8	0.25	1.03	14	28	12.9
9*	0.22	1.024	12.5	25	14.4
10	0.20	1.02	11.4	22.8	15.8

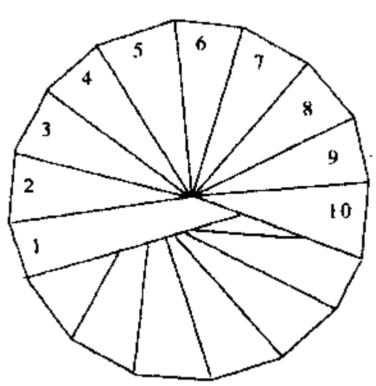


Fig : 1. Rosette with 10 divisions

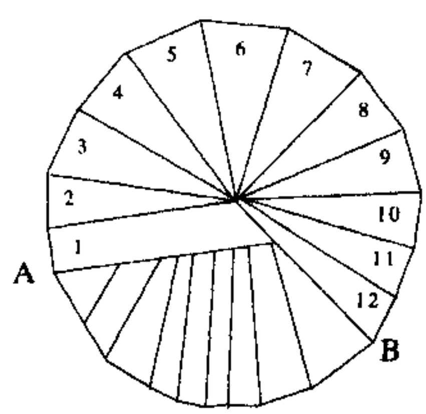


Fig : 2. Rosette with 12 divisions

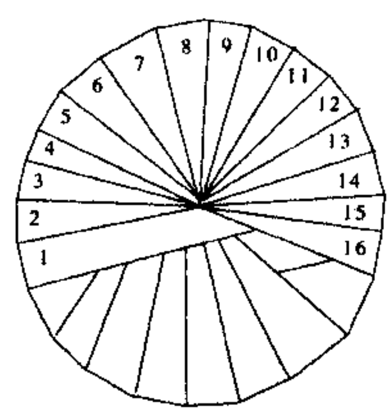


Fig : 3. Rosette with 16 divisions

Finding BO_1 and $\cos \alpha$

Since $BO_1 = 1/2\ BY_1$

$= 1/2 \sqrt{BY^2 + YY_1^2}$

$= 1/2 \sqrt{(0.4)^2 + 1^2}$

$= 0.54$

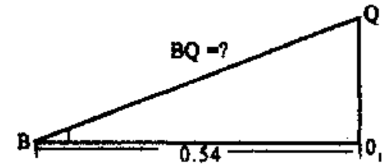


Fig : 8 (b) Finding BQ

and $\alpha = \tan^{-1}\left(\frac{BY}{YY_1}\right) = \tan^{-1}\left(\frac{0.4}{1}\right)$

$= 21.8^0$

$\cos (21.8^0) = 0.928$

The difference of 0.01 in 0.57. Hence theoretical error is less than 2%.

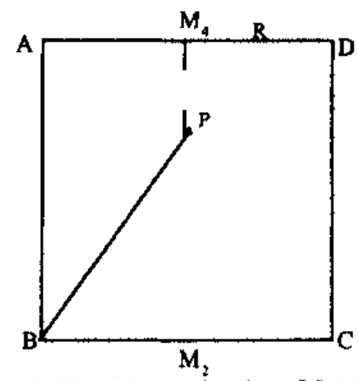


Fig : 9. Marking point P on M_2M_4. BP=BC

Division into 11 equal parts

This simple method divides the square into 11 equal parts with a theoritical accuracy above 99%.

9. Take a square paper and fold perpendicular bisector of side BC only partially near point M_4 as shown.

With B as centre and BC as radius mark only point Pon partially folded perpendicular bisector M_2M_4 as shown. Do not make any creases.

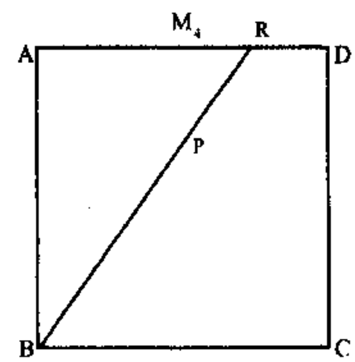


Fig : 10. Extending BP upto point R

10. Make a valley fold through points B&P and let it extend to intersect the line AD at point R.

11. Valley fold RQ as an angle bisector of angle DRB. Q divides DC in proportion of 8:3

Proof is left as an exercise to the folder.

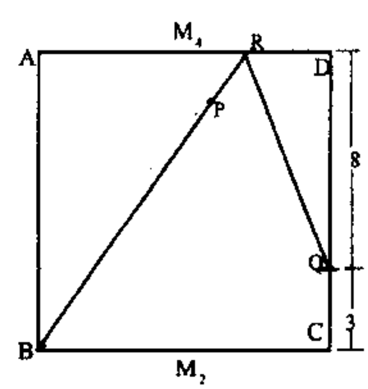


Fig : 11. Angle bisector of $\angle DRB$

5. A represents position 0 (Zero). Y presents position 2. B represents position 5.

Fold the segment AY into 2 equal parts. The mid-point R obtained represents position 1. Further folds, to get the remaining positions 1, 3 and 4 are simple. Thus, the square gets divided into five equal parts. By using binary division method on five equal parts, 10 equal parts are obtained.

Proof : $\tan\theta = \dfrac{M_4A}{AB} = \dfrac{1}{2}$

$\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{3}{5} = \dfrac{YB}{BP}$

Since BP = AB = 1unit

hence YB = 0.6&AY = 0.4 units

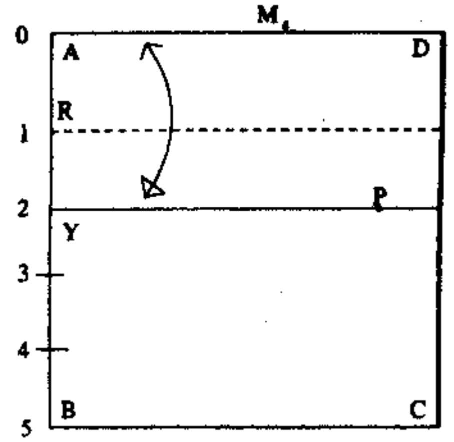


Fig : 5. Division into five equal parts

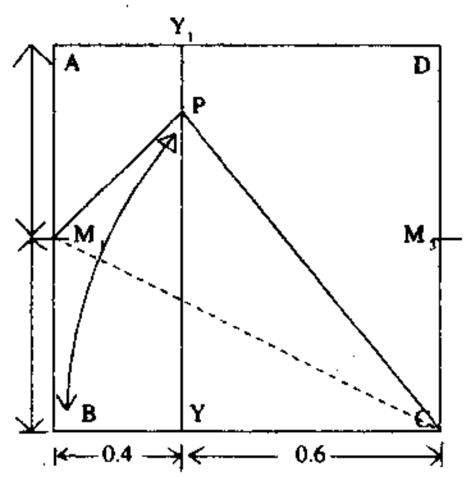


Fig : 6. Division into 2:3 proportion

Division into 7 equal parts

The same method, which is used for 5 divisions, can be used for making seven equal divisions, with addition of one simple fold. Division into fourteen equal parts is further obtained by using the binary method.

6. Mark Y_1 Y using the same procedure as for 5 divisions. Make use of M_1, M_3 and diagonal M_1C as shown.

7. Join B to Y_1 and get perpendicular bisector QT of BY_1. Let the intersection points be Q and T on sides AB and YY_1 respectively. Q divides AB into a proportion of 3:4. Further folds to get positions 1,2,4,5 and 6 are simple.

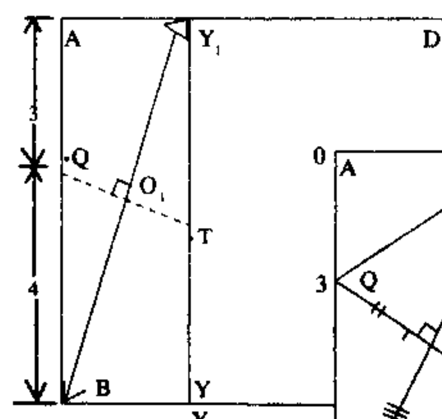


Fig : 7. Perpendicular bisector of diagonal Y_1B

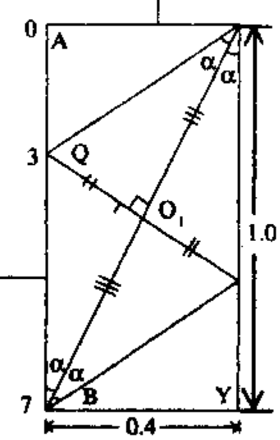


Fig : 8 (a) Finding BY_1 and α

8. a&b Proof : We have to prove

BQ = 4/7 = 0.57

Mark QY_1, BY_1 and BT with a pencil as shown. Let O_1 be the intersecting point of line segments QT and BY_1.

$BQ = \dfrac{BO_1}{\cos\alpha} = \dfrac{0.54}{0.928} = 0.58$

Chapter - 17
Odd Numbered Divisions

Rosettes made with more than eight division offer more amenable resources for wide range of mathematical exercises. They also look more elegant! You can also use them for many other applications. Mostly binary method dividing into two equal part, is used. In addition, trisection method i.e., dividing into three equal parts, is also used, if necessary.

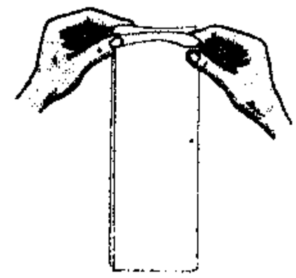


Fig : 1. Trial and error method of trisection

Division into 3 equal parts :

1. Trial and error method : Hold the square little below the curves P and Q as shown. Do not fold. Roll the rounded edges left and right till approximately AP=PQ=QB. Readjust for more accuracy. Pinch and mark the points as P and Q.

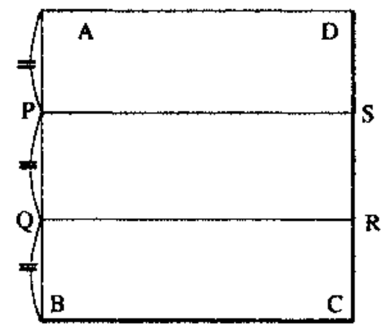


Fig : 2. Square divided into three equal parts

2. Make the creases as shown.

3. By further dividing each part into two equal parts, the square is divided into 6 equal parts. If the Binary Process i.e. `division into two' is further repeated twelve equal parts are obtained. Trial and error method of trisection can be used only in the beginning i.e., we cannot divide the square, further, into 9 equal parts by again using trisection method.

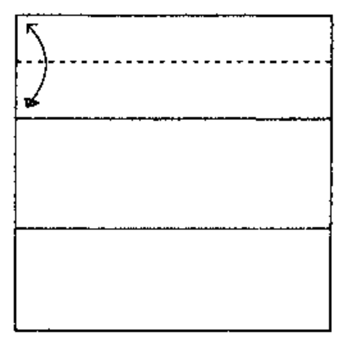


Fig : 3. Further division into six equal parts

Fun-Work : *How will you divide a square into nine equal parts?*

Division into 5 equal parts :

The trial and error method used for three equal parts is not feasible for five equal parts. The method shown here is simple with a theoretical accuracy of 99.8%.

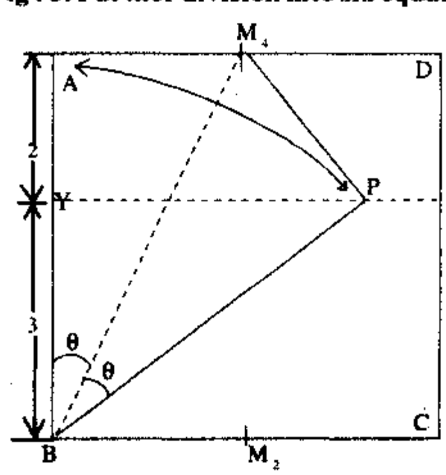


Fig : 4. Division into 2:3 proportion

4. Mark the mid-points M_2 and M_4. Do not crease. Valley fold along BM_4 and mark the point where A meets the square paper at P. Fold the square along a line passing through P and parallel to AD. Let Y be the point where this fold intersects AB. Y divides AB in the ratio 2/3.

6. Angle subtended at the centre by one division, $2d = 28^0$

7. Regular Polygon inscribed :

Since the central angle is 28^0, the corresponding number of sides N in such a polygon would be

$N = 360^0/28^0 = 12.85$ sides

i.e., approximately a 13 sided polygon.

8. Digaonal :

$d^2 = (P_1P_2)^2 + (P_2A_2)^2$

$= (1/4)^2 + (1)^2$

$d = 1.03$ units

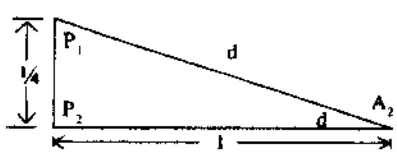


Fig : 8. Measurement of diagonal by Pythagorean method

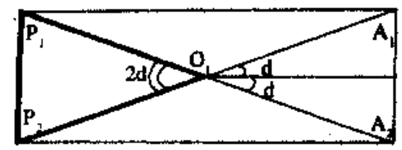


Fig : 6. 2d angle subtended by one division

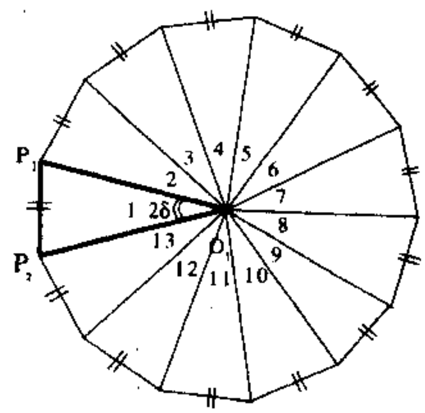


Fig : 7. Regular polygon inscribed by a standard rosette

Chapter - 16
Geometrical Properties of a Rosette

A standard rosette has many useful geometrical properties which change with increase in the number of divisions. Let us discuss a few.

1&2. Take a standard rosette made from 8 divisions. Open it out into a rectangle. Identify all the corners as shown. Cut the upper two divisions of the rosette rectangle.

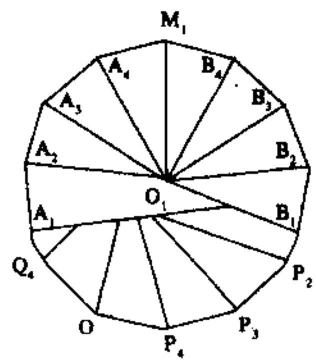


Fig : 1. A standard 8 divisions rosette

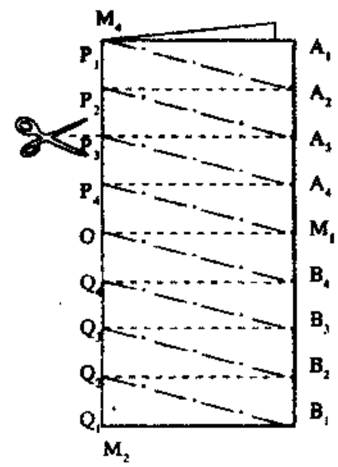


Fig : 2. A standard rosette opened flat

3. Let the angle made by the diagonal P_1A_2 with the side P_1A_1 be **d**. Make valley and mountain folds as shown in the serial order. Note that the position of the line segment P_2A_2 remains same. This part of the rosette contains only 2 divisions, but that is all we need to illustrate the rosette's geometrical properties, since **d** does not change with the size of the rosette.

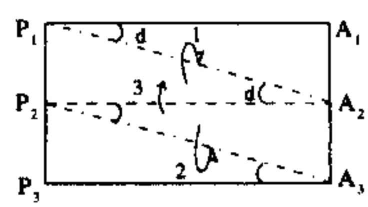


Fig : 3. Two divisions of standard rosette cut and separated

4. Does your two segment folded strip look like this? Fine. Study the transpositions of the lines and formation of angles carefully. Have you noted that angle subtended by each division at the centre is 2d and not d ? Now let us measure some geometrical properties.

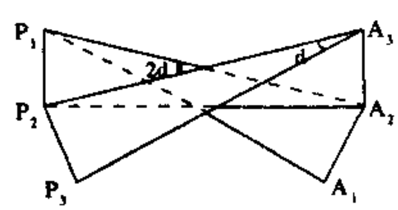


Fig : 4. Two divisions pleated to form a small part of rosette

5. We assume side of the square =2 units.

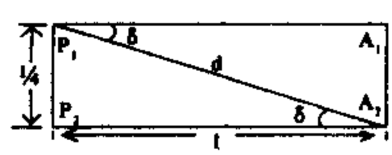
Hence, P_1A_1=1/2 AB = 1 unit,

All measurements are for a standard 8 divisions rosette.

Angle made by the diagonal :

$$\tan d = \frac{P_1P_2}{P_2A_2} = \frac{(1/4)}{1} = 0.25$$

$$d = \tan^{-1}(0.25) = 14^0$$



$$\tan d = \frac{P_1P_2}{P_2A_2}$$

6. Rosettes, when used creatively can help in making of cones.

7. More creatively, double cones can be constructed. Conic sections like parabola, hyperbola etc., can be visualised.

8. A unique application of the rosettes is in making the DNA helix.

9. Rosettes have a pattern which is similar to formatting of floppy disks.

10. Rosettes can exhibit clock design used for primary mathematical exercises.

11. Pie charts for illustrating percentages.

These are few of the applications. There are many more. Now, let us take a look at mathematical properties of rosettes.

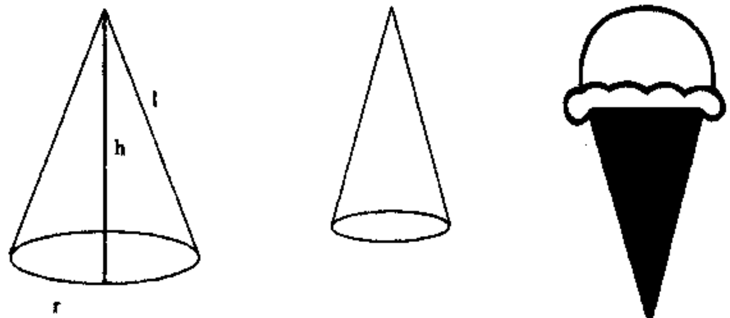


Fig : 6. Right circular cones

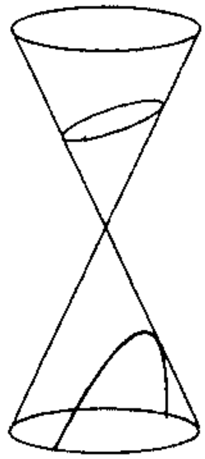


Fig : 7. Conic Sections

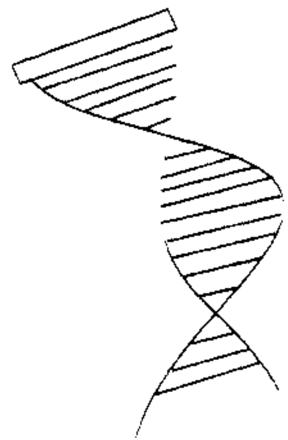


Fig : 8. The DNA Helix

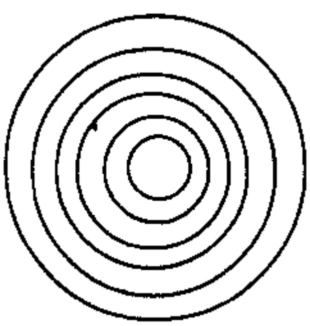


Fig : 9. Sectors and tracks on floppy disks

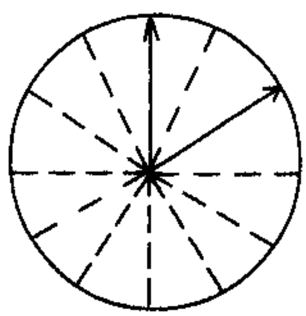


Fig : 10. Clock design

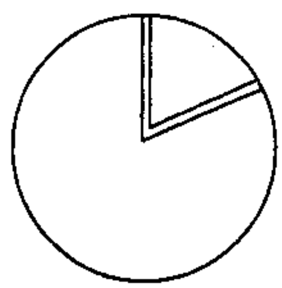


Fig : 11. Percentage Pie Charts

Chapter - 15
Rosettes - Resources of Mathematical Wealth

Rosettes, besides being objects of art, contain lot of mathematical wealth. With rosettes you can feel and see 'mathematics'. An eight-division rosette is sufficient for most of the mathematical exercises done in this book. Nevertheless, much better results are obtained when rosettes are made with more than eight divisions. They have a variety of applications.

List of some applications :

1. Rosette have important parameters relating to properties of circles like arc, chord segment, radius, circumference, area etc. etc.

2. Many theorems on circle can be proved with the help of rosettes.

3. Sine, cosine and tangent waves can be plotted by using suitable rosettes.

4. Rosettes can help in construction of regular polygons.

5. Rosettes can be used for mathematical excercises on right circular cylinders.

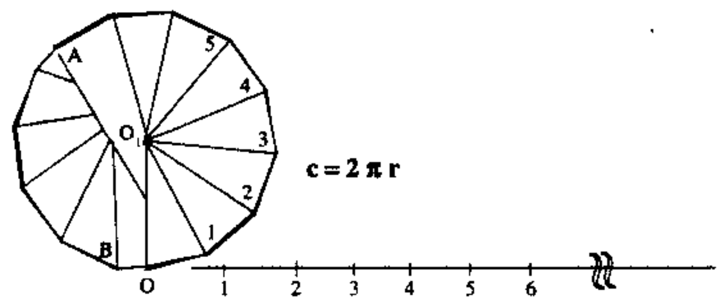


Fig : 1. Circumference of a circle

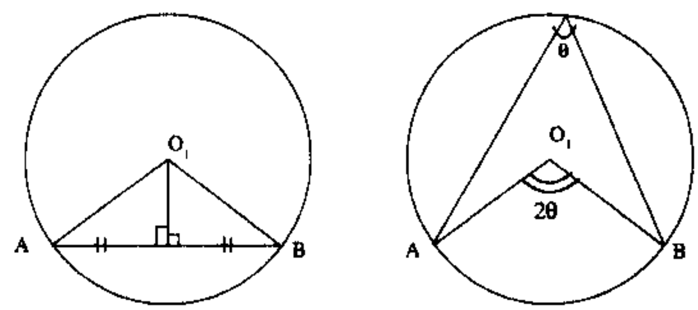


Fig : 2. Theorems on circle

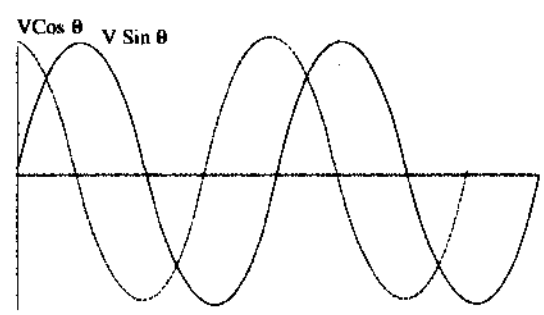


Fig : 3. Sine-Cosine waves

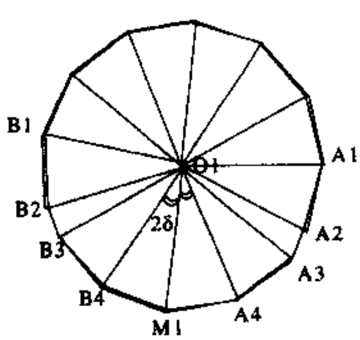


Fig : 4. Regular Polygons

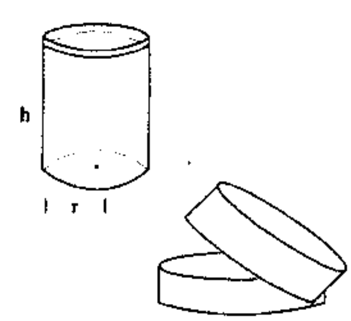


Fig : 5. Cylinders

Part III
The Mathematics

Chapter - 14
Infinite Patterns

Do you know how many designs can be created in a kaleidoscope? Virtually infinite ! The time required to see all the patterns at a rate of 1/2 minute per pattern would run into thousands and thousands of years ! Similarly you can draw infinite designs within a circle. Is this possible with a rosette ?

In Kaleidoscopes, coloured glass pieces can criss-cross, interleave overlap on one another randomly. In rosettes, random juxtaposing of front and back coloured portions of the same square is not possible. But there are many more possibilities. You have already seen some possibilities in various rosettes. Some more random designs are given below.

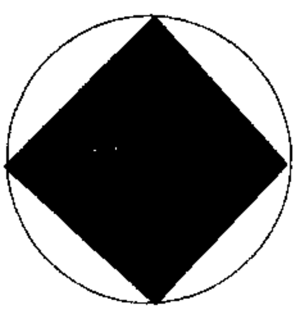
The following clues/rules should help you in making these patterns.

(a) Symmetry and a-symmetry should be judiciously used.

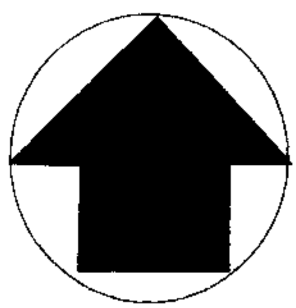
(b) If you have any difficulty then draw the design on a plane white rosette and open it flat and study the pattern carefully.

(c) You can use proportionate strips but avoid cutting, pasting etc. If necessary, use more than one square.

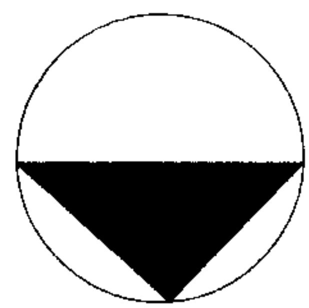
Good Luck!



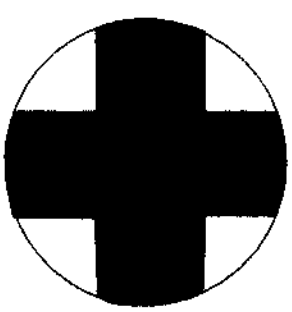
1. Square Pegs in Round Holes



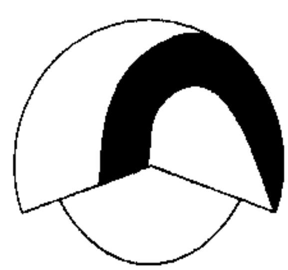
2. Shelter



3. Family Planning



4. Red Cross



5. Space Shell

4. Does the reverse side look like this? Good. Now make the standard rosette creases and fold the rosette in the usual way.

5. A Dweep-Jyoti reosette has been created.

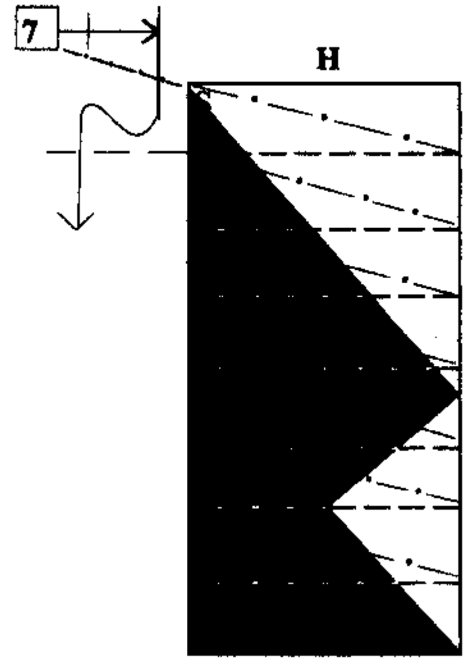


Fig: Q 4. Standard rosette creasing pattern

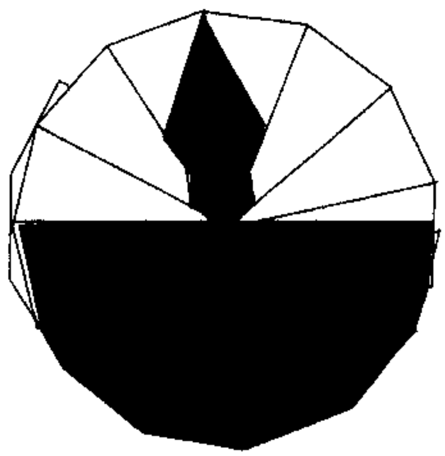


Fig: 5. Dweep-Jyoti rosette completed

Fun-Work : *Make a double Dweep-Jyoti rosette.*

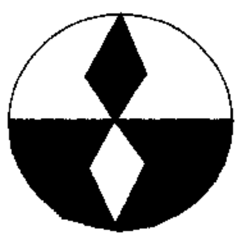


Fig: Q 6. Double Dweep-Jyotirosette

Chapter - 13
Dweep-Jyoti Rosette

Make this rosette with colour paper. This rosette invovles a lot of geometry, so bring a pencil and a ruler. Mark lines AC and $M_1 M_3$ lightly with the pencil on your square, alternatively, you can fold these lines, lightly.

1a. Find the points P,H,S,G,E &F on the square such that:

$M_1E=EF=FM_3= 1/3\ M_1M_3$ and

$M_1G=GE=1/2\ M_1E= 1/6\ M_1M_3$

b. Draw a line GI such that GI is perpendicular to M_1E

c. H is the intersection of GI and AC. Draw a line PS which is perpendicular to AC and passes through point H.

2. Draw lines PQ and SR parallel to AC. Join Q and R. Our rectangle is PQRS. Make the folds in the order shown.

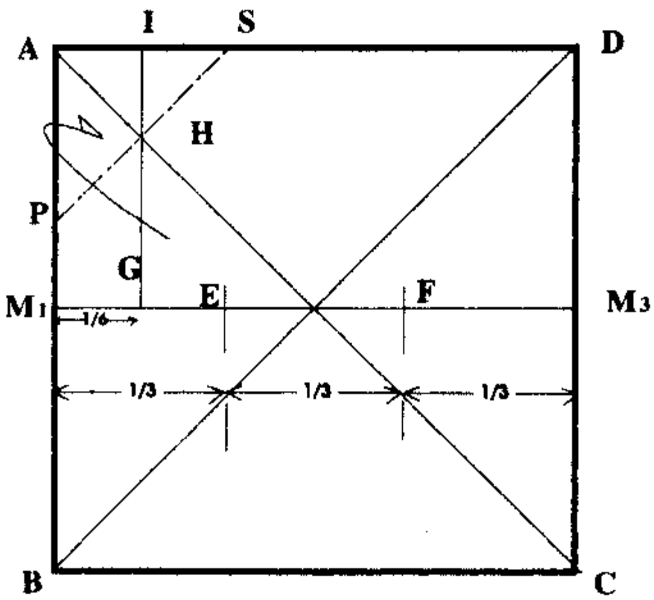


Fig: 1. 1/3 & 1/6 Division on median and marking the required points

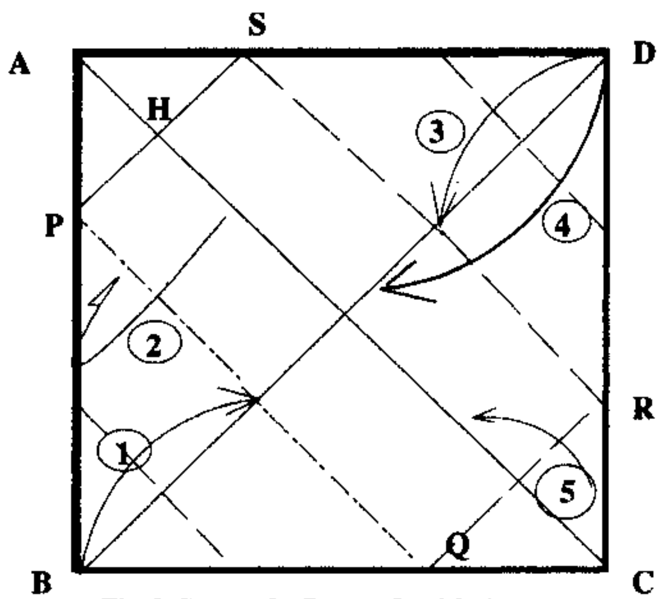


Fig: 2. Creases for Dweep-Jyoti design

3. Does the model look like this now? Turn the paper over as shown from left to right.

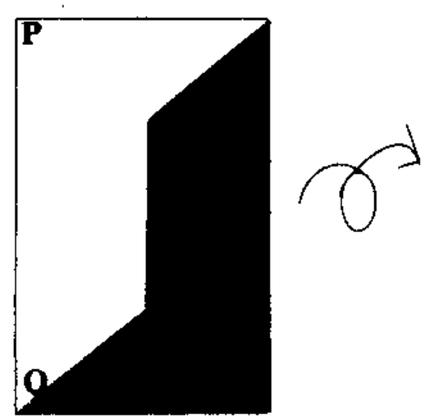


Fig: 3. Back side of rosette element for Dweep-Jyoti

Chapter - 12
Sun - Moon Rosette

Many origami books contain diagrams which are very brief. In such cases you have to work by trial and error to start with, until you become an expert folder. Here is some practice. Have fun and do not lose heart.

1. Fold along the lines as shown.

2. Does your rectangle look like this? Good. Now make the standard rosette crease and fold the rosette, pleat by pleat, in a serial order. Tuck away extra flaps.

3. A sun-moon rosette has been created.

***Fun-Work** (1). Try to make a rosette as shown. Use more than 8 divisions for smooth curvs.*

(2). How will you increase and decrease diameters of inside concentric circles independently ?

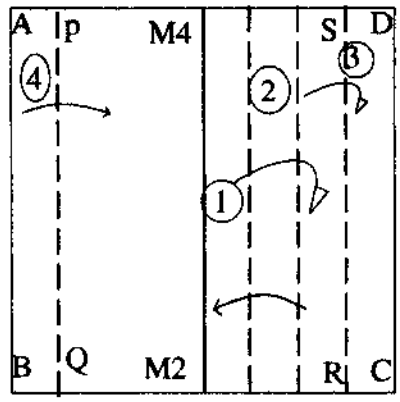


Fig: 1. Required pattern of creases

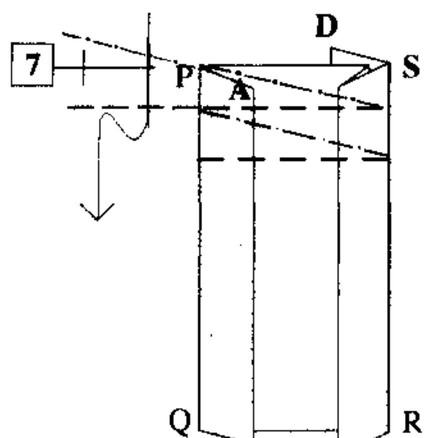


Fig: 2. Standard rosette folds

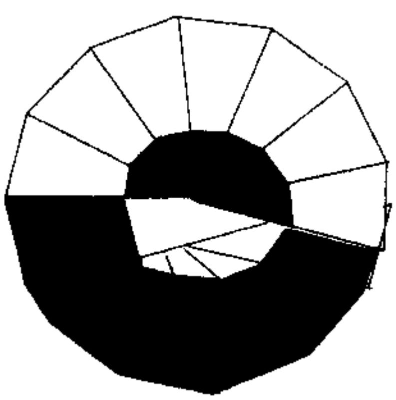
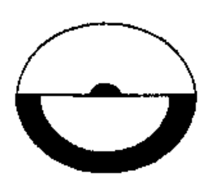


Fig: 3. Sun-Moon rosette completed

F/W 1



F/W 2

Chapter - 11
Vice President Rosette

Make this rosette with a colour paper. Remember to make sharp and firm creases. Final rectangle must have same proportion of 2x1.

1. Take $d_1=d_2=d_3$ = about 1/16 of the side of the square ABCD and fold the square as shown. The number on the arrows indicates the sequence of making folds.

2. Does your rectangle look like this ? lovely. Lay it flat.

3. Now make standard rosette creases and fold the rosette pleat by pleat in the usual way.

4. Vice president rosette has been created.

Fun-Work : Can you make a President Rosette?

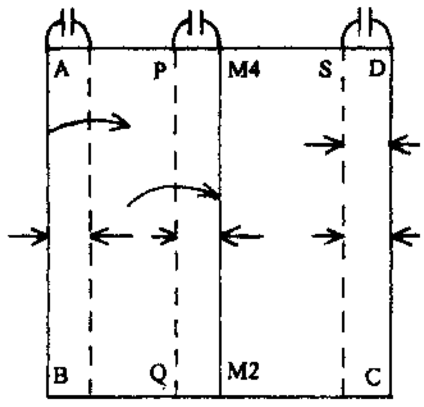


Fig: 1. Basic pattern of folds

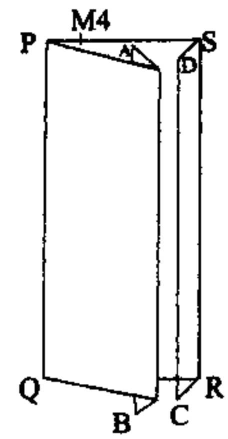


Fig: 2. Rosette element with sandwiched white strip

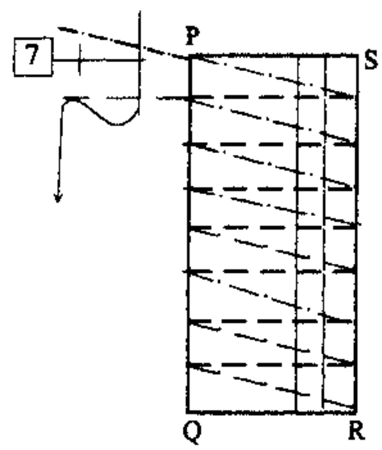


Fig: 3. Standard Rosette folds

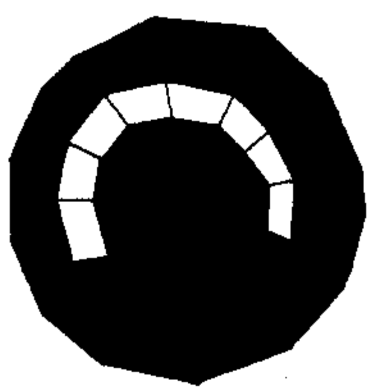
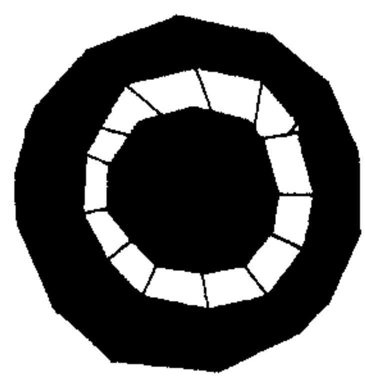


Fig: 4. Vice President Rosette completed



F/W President Rosette

Chapter - 10
Half Spiral Rosette

This rosette needs a colour paper. Make sharp and accurate creases with the back of your thumb nail. Lay the paper against a flat and hard surface. Press firmly.

1. Book fold a square along its vertical median M_2M_4.

2. Valley fold small slant strip from point A on the front flap as shown.

3. Make a standard rosette crease and fold the rosette in the usual way.

4. A half spiral rosette has been created.

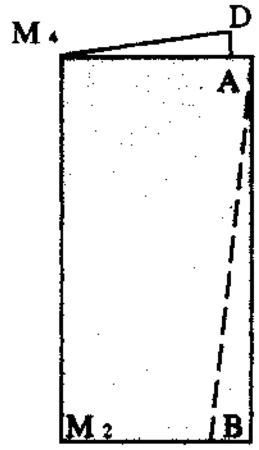


Fig: 1. Book fold square

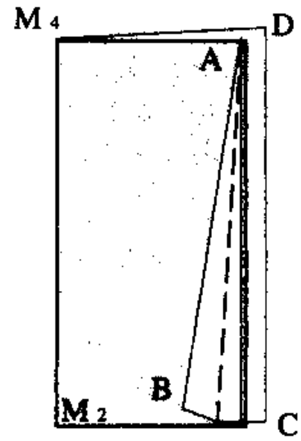


Fig: 2. Small slant strip folded out

***Fun-Work :** (1) & (2)*

1. Can you make two half spirals rosette?

2. And a full spiral rosette?

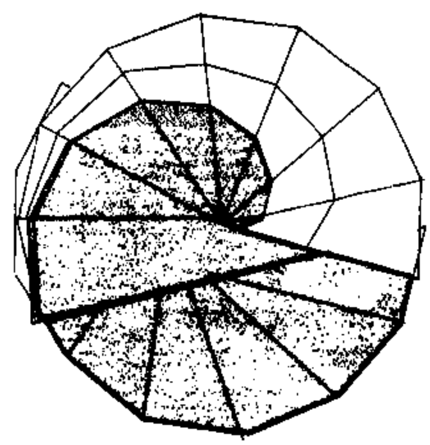


Fig: 2. Half spiral rosette

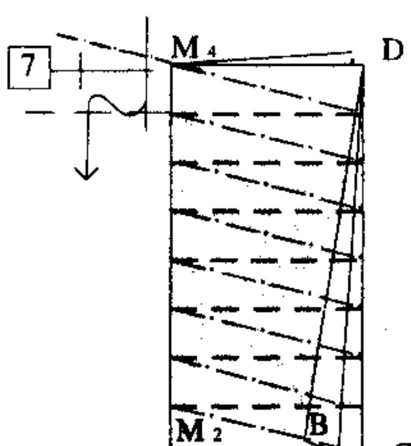
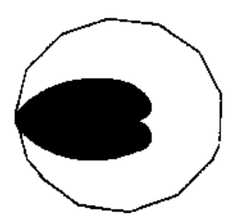


Fig: 3. Standard rosette folds



F/W 1

F/W 2

Chapter - 9
Rising Sun Rosette

Make this rosette with a colour paper. Red fluroscent paper is ideal. Remember to make sharp and firm creases.

1. Bookfold a square into half and lay it flat.

2. Valley fold a small strip on the front flap as shown.

3. Make the standard rosette creases and fold the rosette in the usual way. Transpose corners M_4 and C.

4. A rising sun rosette has been created.

***Fun-Work** : Can you make a full rising sun rosette?*

Fig: 1. Book folded square

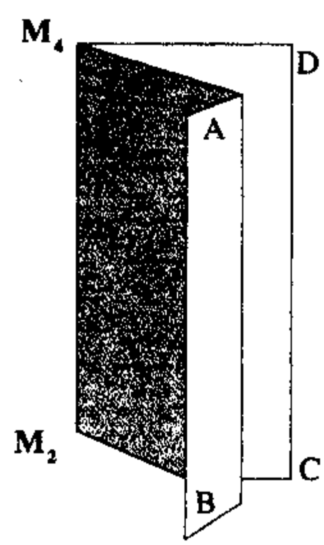


Fig: 2. A small strip folded out

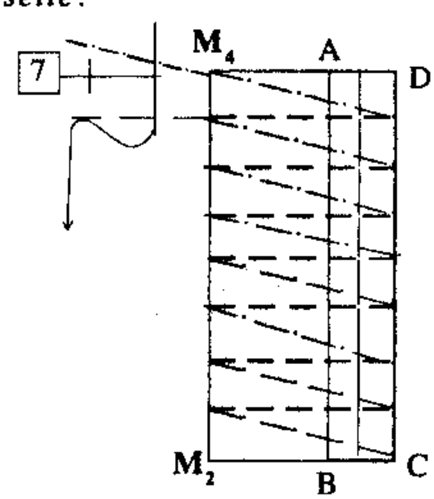


Fig: 3. Standard rosette creases on modified element

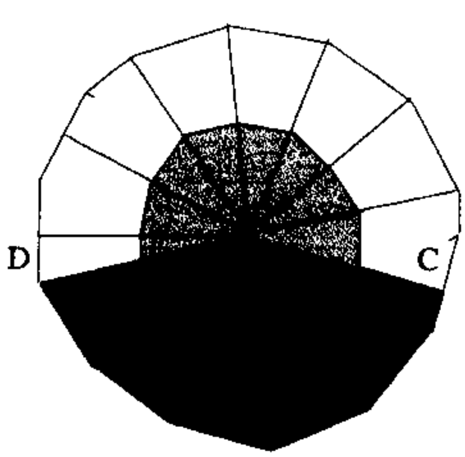
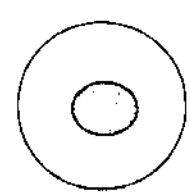


Fig: 4. Rising Sun rosette



F/W

Chapter - 8
Inside - Out Rosette

Make this rosette with designed or glazed paper. Make sharp creases with the back of your thumb nail. Lay the square against a flat hard surface. Press firmly and see that the diagonals cut the corners exactly.

1. Bookfold the square into half and lay it flat. Make standard rosette creases on this rectangle. Now open out the rectangle along the central vertical line M_4M_2 and continue folding until it again becomes a complete bookfold but from inside out.

2.&3. Now fold this `colour changed' rectangle in the usual way, pleat by pleat, until an inside-out rosette has been created.

Fun-Work: *You can try out usual modificaitons (eg. Sun - moon, Half-president, Dweep - jyoti etc.) with Inside-out rosette. Do you get colour inversion effect?*

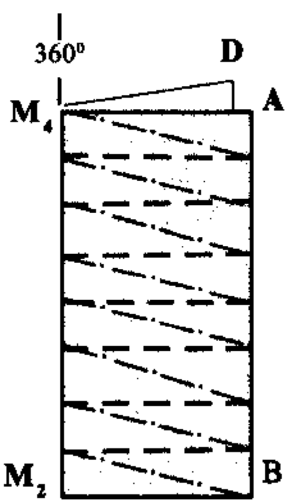


Fig: 1. Standard rosette element

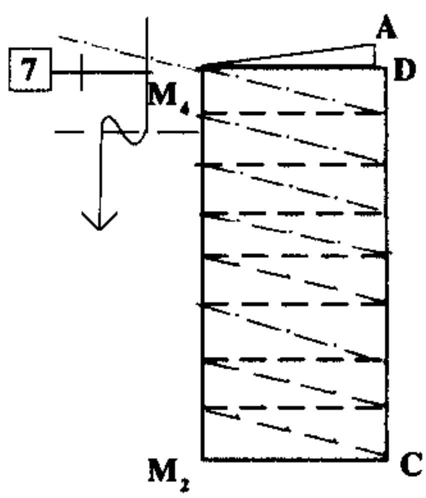


Fig: 2. Rosette elements turned inside out

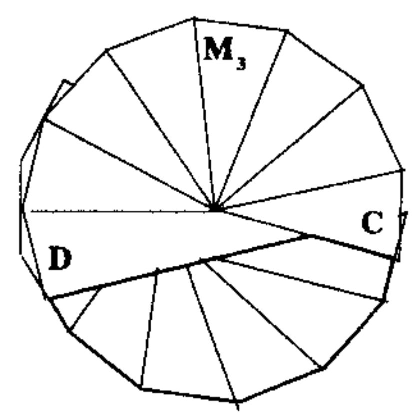


Fig: 3. White coloured rosette

Chapter - 7
Single Element Rosette

This rosette is made out of the standard rosette, which you have already mastered. A colour paper is suitable. You can make a single element rosette from any rectangle of proportion 2x1 or from a long rectangular strip.

1. Take a standard rosette and stretch it flat. Open the bookfold.

2. You will get original square ABCD. Left rectangle ABM_2M_4 is a mirror image of rectangle M_4M_2CD. Note the change in valley and mountain folds. Cut the square through M_2M_4. Take rectangle M_4M_2CD.

3. This rectangle has all creases. Start folding it pleat by pleat in the usual way.

4. A single element rosette has been created.

Fun Work : *Take a rectangular strip of a design paper and make a greeting card as shown.*

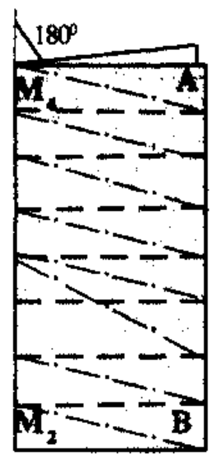


Fig: 1. Standard rosette flattened

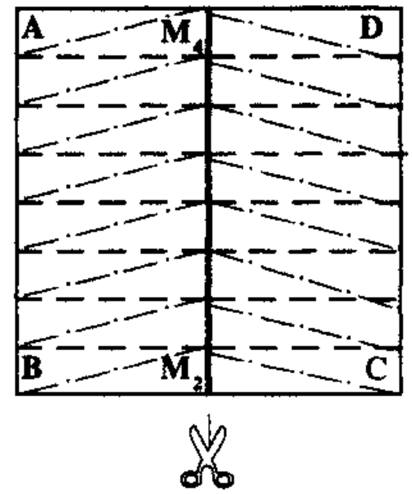


Fig: 2. Standard rosette cut into two elements

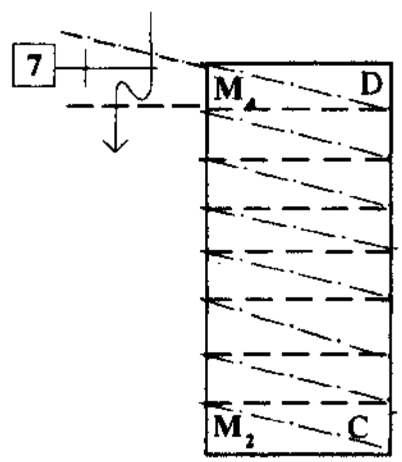


Fig: 3. Single layer flattened rosette element

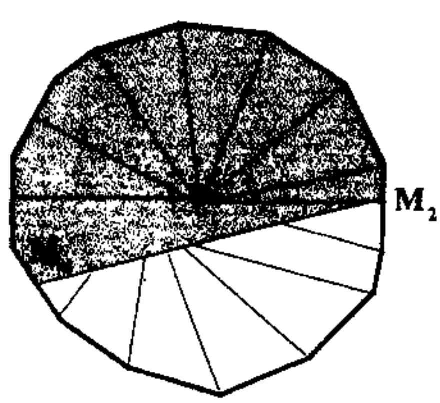
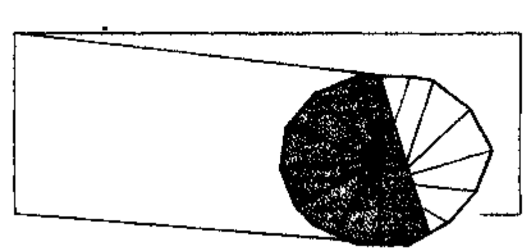


Fig: 4. Single element rosette completed



F/W . Greeting card from a rectangular strip

Chapter -6
Standard Rosette Separated

Take a standard rosette made from a colour paper. Check for sharp creases. If necessary repeat the creases.

1. Stretch the rosette into a flat rectangle and open the bookfold. You will get two rectangles in tandem, each having standard rosette creases.

2. &3. Start pleating from the top of the right rectangle and form a rosette from this single layer. The left hand rectangle will automatically form a helical staircase like structure. Pleat the staircase structure into a rosette.

4. A separate rosette is created. The two single rosettes are joined by the arc M_2M_4.

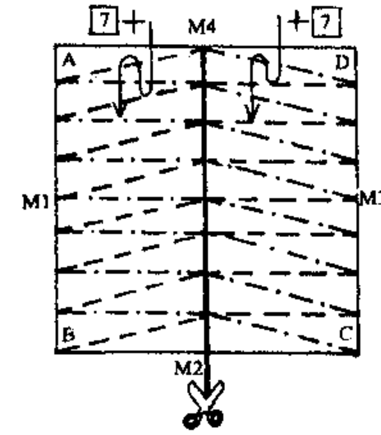


Fig: 1. Standard rosette opened flat

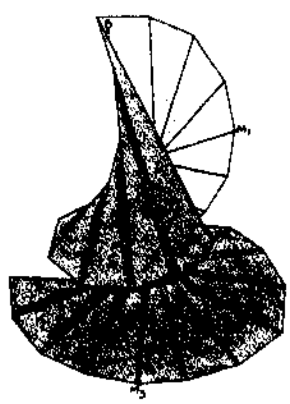


Fig: 2. Right element pleat folded

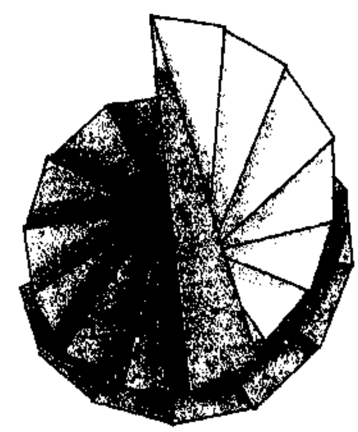


Fig: 3. Left element pleat folded on right element rosette

Fun Work 1: *Take a three layered rectangle and make standard rosette creases. Open the layers and form a trio of rosettes. Can you increase the number of layers ?*

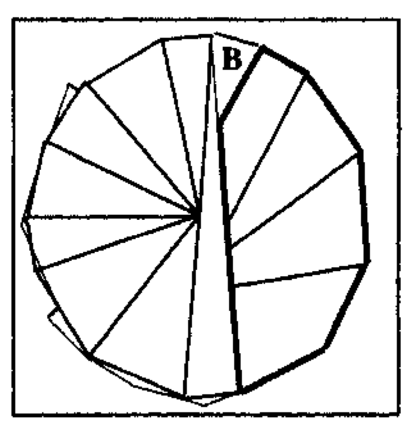


Fig: 4. Standard rosette separated

Phases of Making a Rosette

In making a rosette we go through three main phases.

20. Phase I : Involves making a rectangle of 2 units x 1 unit whether by cutting or folding; single, double or multiple layered. And dividing it into 8 equal parts by horizontal valley folds.

21. Phase II : Involves making the mountain creases that have been elaborated for the standard rosette. These creases being the same for all rosettes will be henceforth subsequently referred to as the **'Standard rosette creases'** and 2x1 pre-creased rectangles 'standard rosette element.'

22. Phase III: Involves making a series of pleat folds as shown in the standard rosette. The paper appears to twist while pleating. Hence we will refer to these folds as the **'Rosette pleat folds'**.

23. Rosette is completed after the three phases.

Even though various rosettes go through the same phases they attain wonderfully different designs.

Now let us enter the world of rosettes.

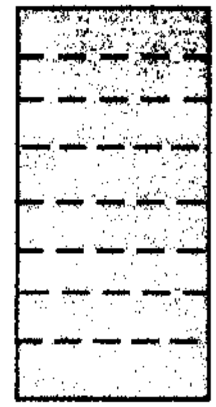


Fig: 20. Phase I. Eight equal valley folds

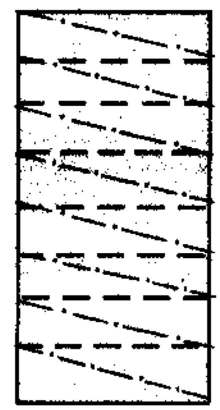


Fig: 21. Phase II. Diagonal mountain folds

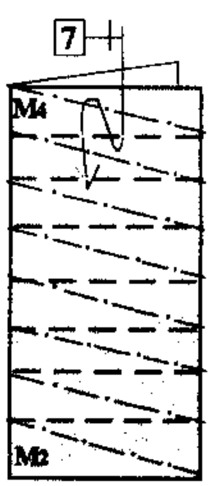


Fig: 22. Phase III. Pleating

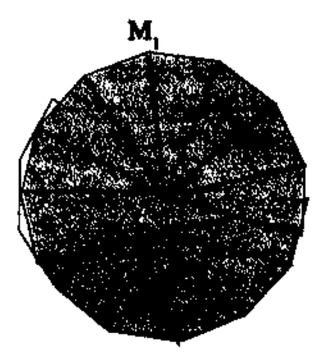


Fig: 23. Rosette completed

12. By now we ought to have seven horizontal valley creases parallel to the shorter side of the rectangle and 8 mountain creases along the diagonal of each division.

13. (a) &(b) Valley fold the upper most division down and then try valley fold along the diagonal of this division.

14. One pleat fold is completed. Second pleating continued; do not unfold first mountain crease. Valley fold along the second division in such a way that the second rectangle faces you. (Fig. 15).

15. Now, simultaneously, valley fold along the second diagonal and mountain fold along the first horizontal division. (Note reversal of folds from back side). This will complete second pleat fold.

16&17. Continue for next pleats in serial order. The rosette is beginning to take shape.

18. Pleat folding completed. Transpose corners M_4 & B.

19. The arc AB is in full view. We have now created a standard rosette

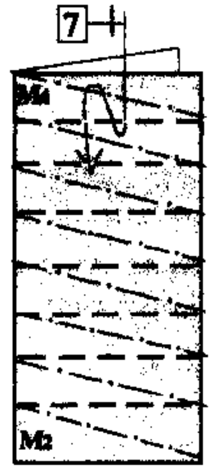


Fig: 12. Sets of valley & mountain folds

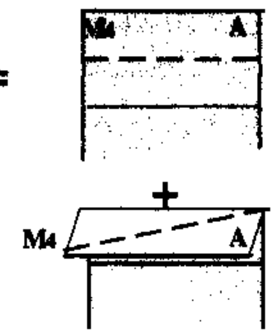


Fig: 13. (a) & (b) First step in pleating

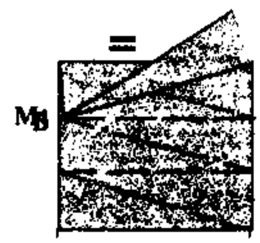


Fig: 14. One pleat fold completed

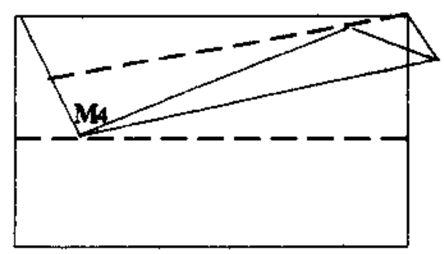


Fig: 15. Pleat folding continued

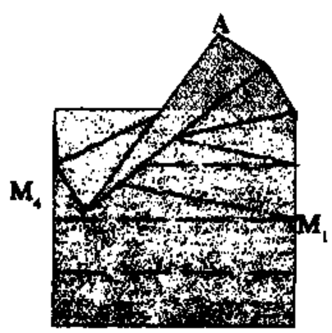


Fig: 16. Pleat folding & beginning of formation of rosette

Fig: 17. Formation of rosette in progress

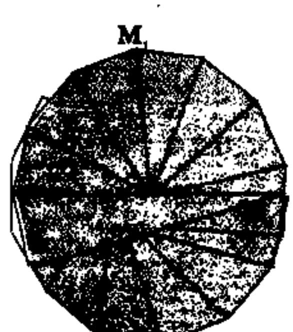


Fig: 18. Rosette formation completed

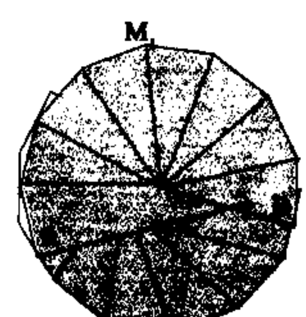


Fig: 19. Arc AB in full view

8.(a) & (b) The next step is to mountain fold-unfold along the 8 parallel diagonals of the 8 equal rectangular divisions. Your folds ought to run diagonally from corner to corner. If there is an error, try again.

Since making valley creases is simple and accurate continue as follows:

9. Valley fold the uppermost division down so that its reverse side comes in front.

10. Valley fold accurately along the diagonal of this division. Unfold the paper back into a rectangle and notice that a mountain crease has been formed in the first division.

11.(a)&(b) In the same manner make a mountain crease in the second division. Continue the same process for remaining divisions.

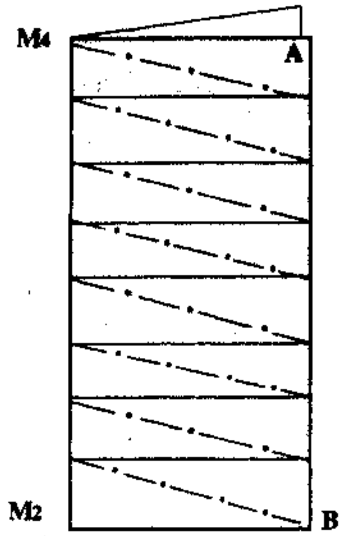


Fig: 8. Diagonal mountain folds

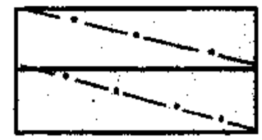


Fig: 8. (a) No Error

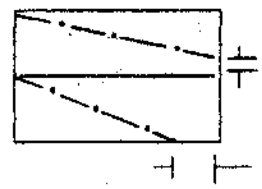


Fig: 8. (b) Error

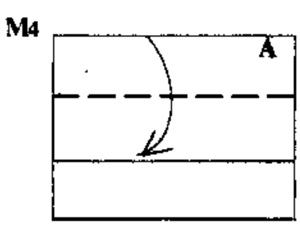


Fig: 9. Reversing one divison ....

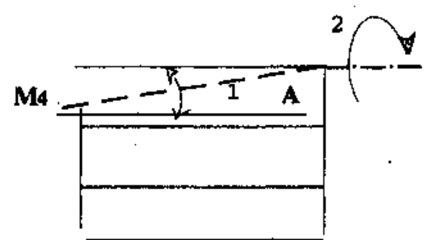


Fig: 10. .... to make mountain fold

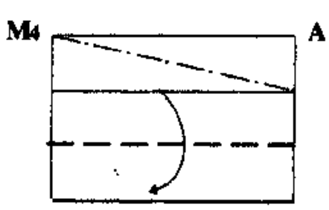


Fig: 11. (a) Mountain creases by reversing the strip

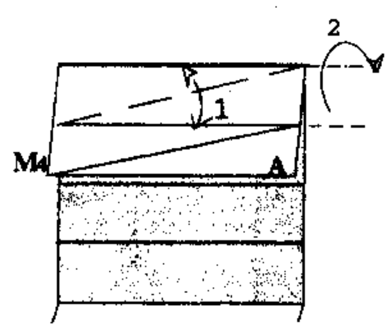


Fig: 11. (b) Mountain creases continued

Chapter - 5
Standard Rosette

All exercises in this book make use of standard rosettes. Hence it is advisable to master the technique fully. The technique shown here is elaborate and with full details. However, the folder can evolve his/her own convenient technique after grasping the prescribed method.

1(a)&(b) Book fold a square paper in half. Two equal rectangles of proportion 2x1 are obtained. The same proportion is used in further exercises.

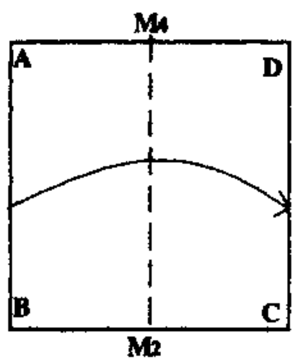
2. The first step is to divide the rectangel exactly into 8 equal horizontal divisions by making valley creases as shown. Do not fold over and over again to get 8 equal divisions. Use binary method as discussed below.

Binary method

3&4. The two layered rectangle is divided in half by joining opposite edges. Two squares are formed. Each square is again divided in half by valley folds.

5&6. Join the botton edge M_2 B to the median of upper square M_4OM_1 A. Repeat the same procedure in the other half. Finally the top and bottom rectangles are valley folded.

7. The rectangle is now divided exactly into 8 equal parts.



a
Fig: 1. (a) Book fold

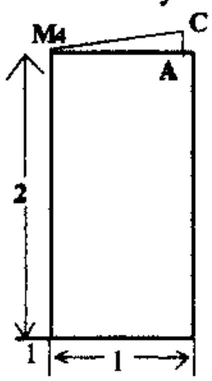


Fig: 1. (b) Book fold completed

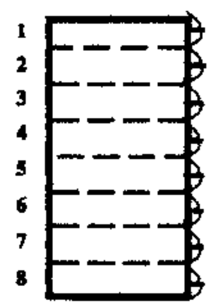


Fig: 2. Eight equal divisions

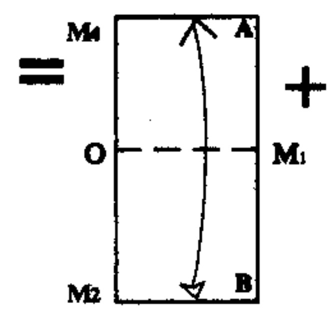


Fig: 3. Binary method of division

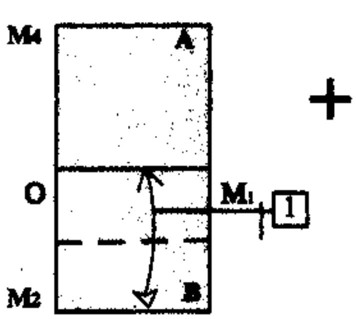


Fig: 4. Binary method continued

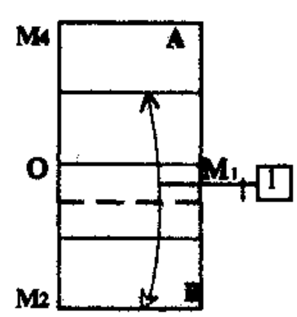


Fig: 5. Binary method continued

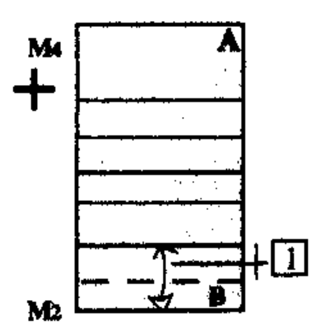


Fig: 6. Binary method continued

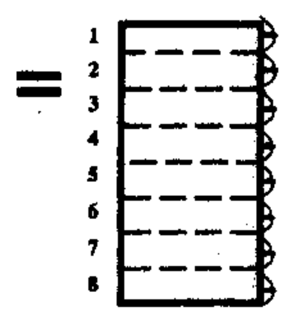


Fig: 7. Eight equal division continued

Part - II
The Aesthetics

5&6. Turn Over

5. Turn over from side to side.

6. Turn over from top to bottom.

Fig: 5. (a) Turn over from side to side

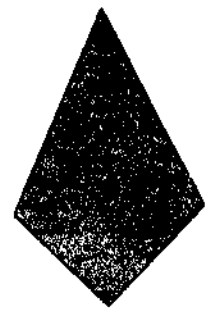


Fig: 5. (b) ....like this

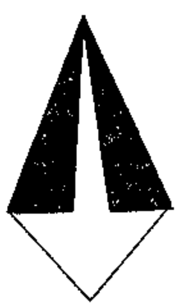


Fig: 6. (a) Turn over from top to bottom

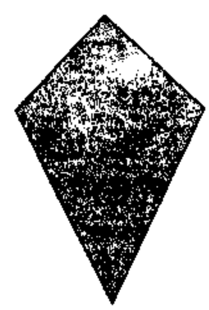


Fig: 6. (b) ....like this

3. Pleat Fold

New symbol 

Old symbol 

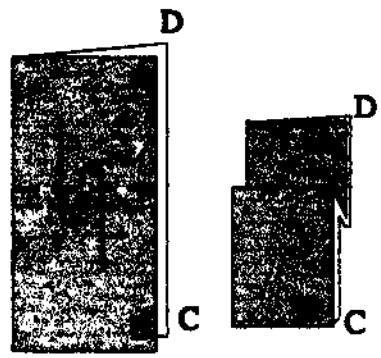


Fig: 3. (a) Pleat folding: (parallel pleats)

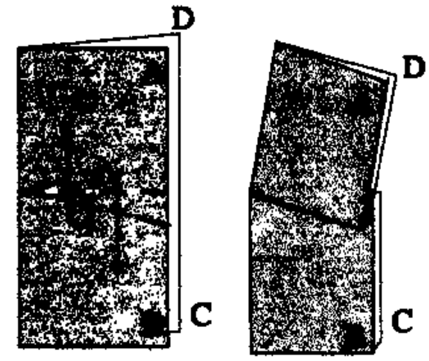
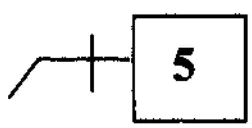


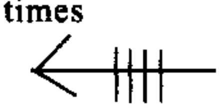
Fig: 3. (b) Pleat folding; (intersection pleats)

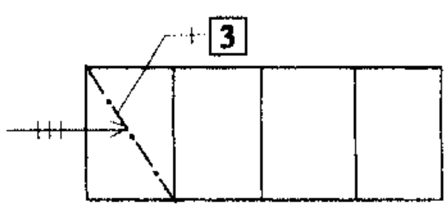
Pleat fold describes closely placed combination of valley and mountain folds. Most of the models in this book use pleat folding as shown in Fig 3(b).

4. Repeat Fold

New symbol
Repeat five times 

Old symbol
Repeat four times





Repeat fold symbol describes the number of repetitions to be performed. The number inside the box in the new symbol and the number of small lines intersecting the arrow stem in the old symbol describe number of repetitions to be performed.

Fig: 4. (a) Any of this repeat symbol means:

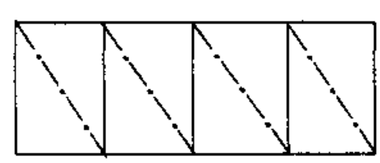


Fig: 4. (b)...... repeat three times like this

2. Mountain Folds

A mountain fold line is of a dash-dot type or dash-dot-dot type with a curved arrow having a half hollow head, indicating the direction of the fold.

As you might have guessed by now valley fold causes the paper to appear like a valley as viewed from the front and a mountain fold is just a valley fold in reverse.

Since a valley fold is simpler to make than a mountain fold, it is wiser to reverse the paper and make a valley fold, rather than make a mountain fold, wherever it is possible. Can you guess how mountain creases are done?

You should have a good practice of making a criss-cross of valley and mountain folds. Neglecting the difference between valley and mountain folds is as serious as not discriminating between + and – symbols in mathematical operations.

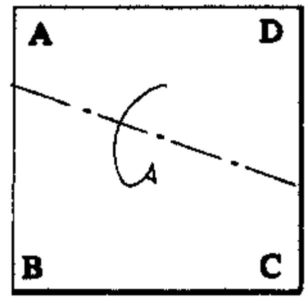
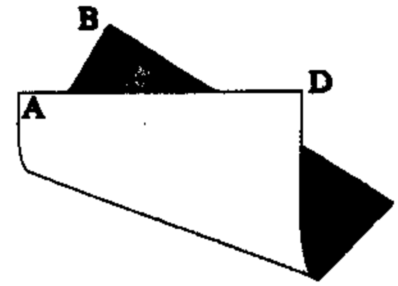


Fig: 2. (a) Symbol : mountain fold in down ward direction



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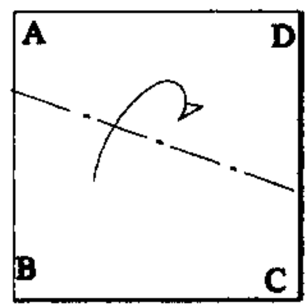
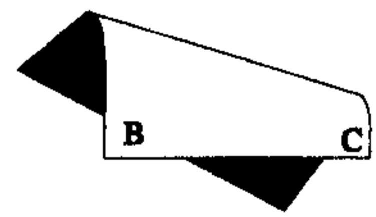


Fig: 2. (b) Symbol : mountain fold in upward direction



.......like this.

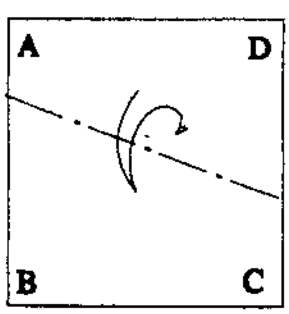
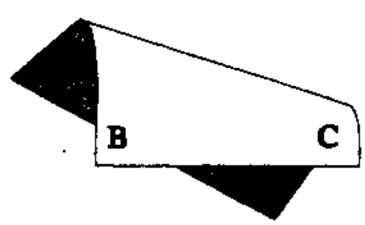


Fig: 2. (c) Symbol : mountain fold and unfold



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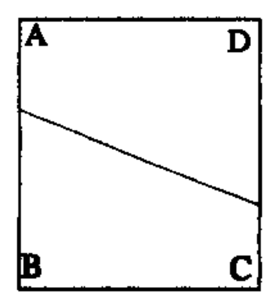


.......open again

Chapter - 4
Symbols Used

Standard symbols are used in this book to explain the folding procedure for origami objects. Most of the symbols are taken from the diagramming panel at `Convention 88' held in New York. The panel, consisting of internationally renowned folders, worked on the topic: 'Diagramming in search of uniformity'.

1. Valley Folds

1. (a) & (b) Valley fold is of a dash-dash type with a curved arrow indicating the direction of the fold.

1.(c) A special double headed arrow indicates fold & unfold operations. Valley folding & unfolding is done in order to form valley creases for reference lines on your square paper.

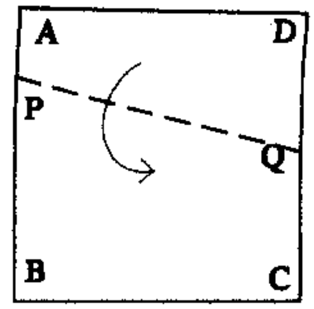
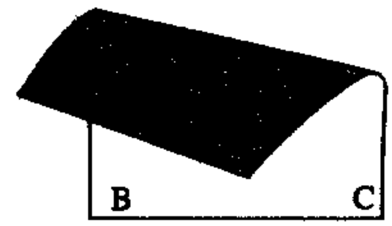


Fig: 1. (a) Symbol : Valley fold in downward direction



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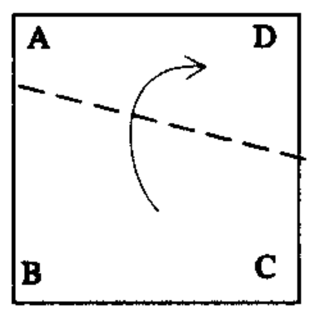

Fig: 1. (b) Valley folding in up ward direction

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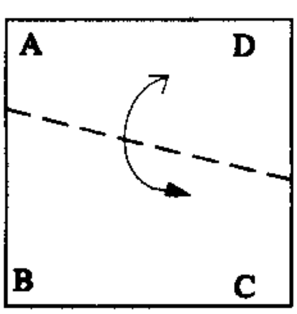
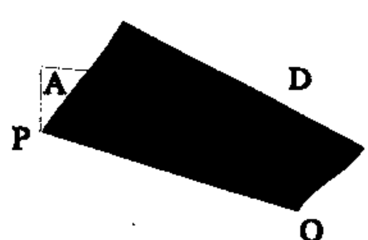


Fig: 1. (c) Symbol: Valley fold and unfold



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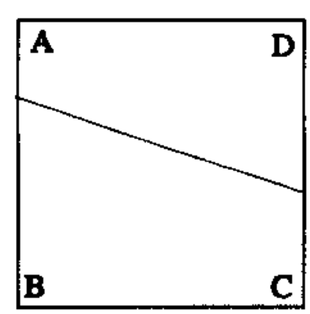


.......and open again.

Chapter - 3
The Folder's Square

Who is a folder?

A folder is the person who folds a square piece of paper, without cutting or pasting, in order to make an origami object.

A square is just a square paper - is it not?

Yes, an origami "square" is just a square piece of paper, but not quite so. It is much more than that. For a folder, a square is a means of creating many, many wonderful origami objects.

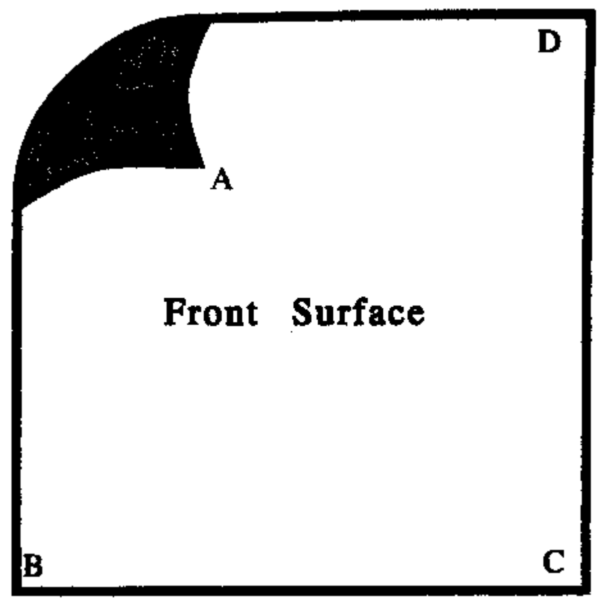


Fig: 1. Two surface of a square

1. The square paper shown here is assumed to have a white front surface and a darker back surface. Any paper that you have, whether it is coloured, glazed, tinted or designed, will almost always have different surface qualities for its back and front sides.

2. When you practice the models described in this book, always put white, plain or rough surface of the square in front of you to begin with. Also follow the points A,B,C, D,M_1,M_2,M_3, and M_4 as shown. M_1,M_2,M_3 and M_4 are mid-points of the respective sides and O is the centre.

Not doing so would land you in an unknown city without having a road map. It is possible but difficult to reach your destination in such a situation.

Hence, in any model, mapping at least the corner points A,B,C and D is very essential.

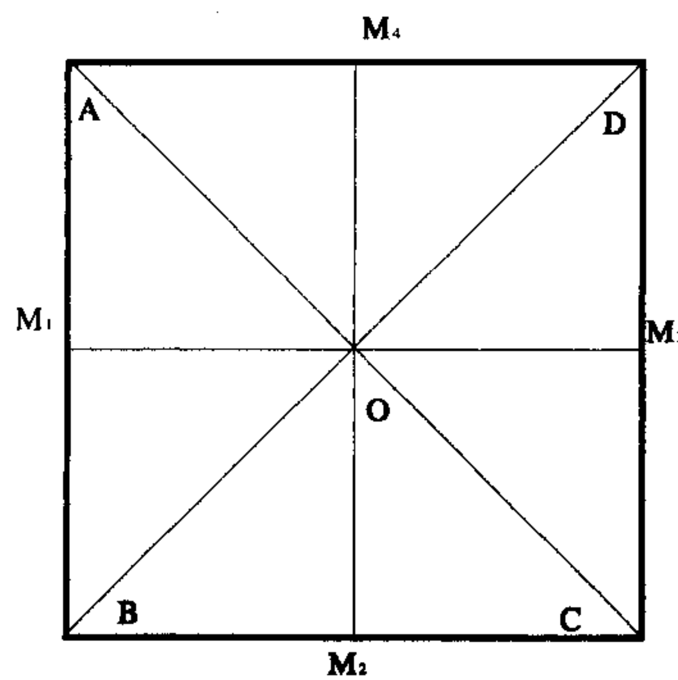


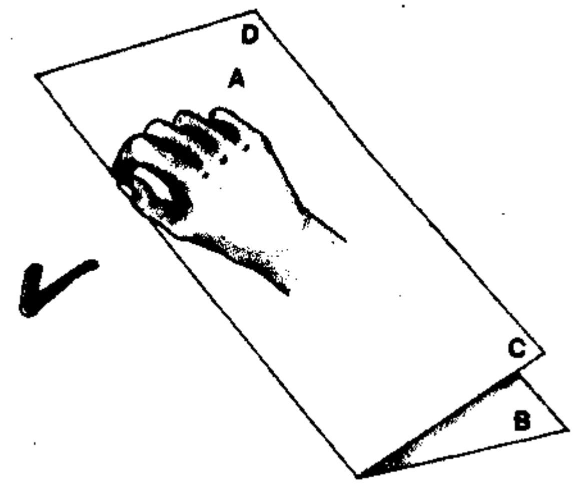
Fig: 2. Mapping the square with corners, mid-points and the centre

Chapter - 2
Before You Start Folding

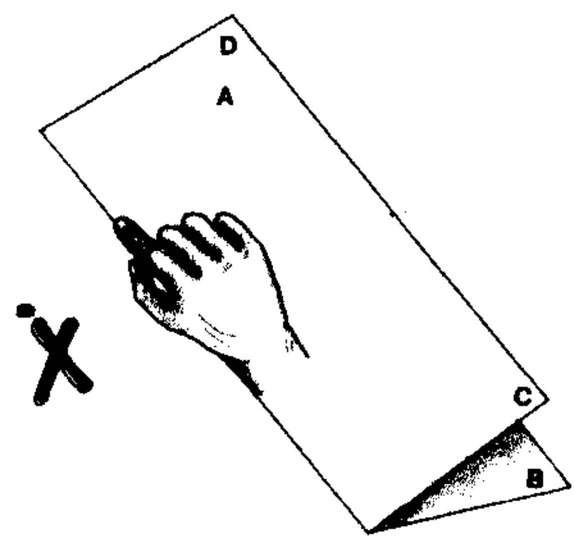
All the models shown in this book require sharp and razor edged creases. To get them you will need a hard and flat surface. Make creases by pressing firmly with the inverted thumb nail. Folding with fingers distribute the pressure resulting in blunt creases. So never fold with the fingers and never fold in the air.

However, in many animal models folding is done specifically in the air and with the fingers. Also to avoid creasing altogether, a very special technique called wet folding is used. It employs a wet cloth to get curves.

For these models, you have to make sharp and firm creases. The pressure is concentrated at only one point when you fold with a finger nail (....tangent to a circle).

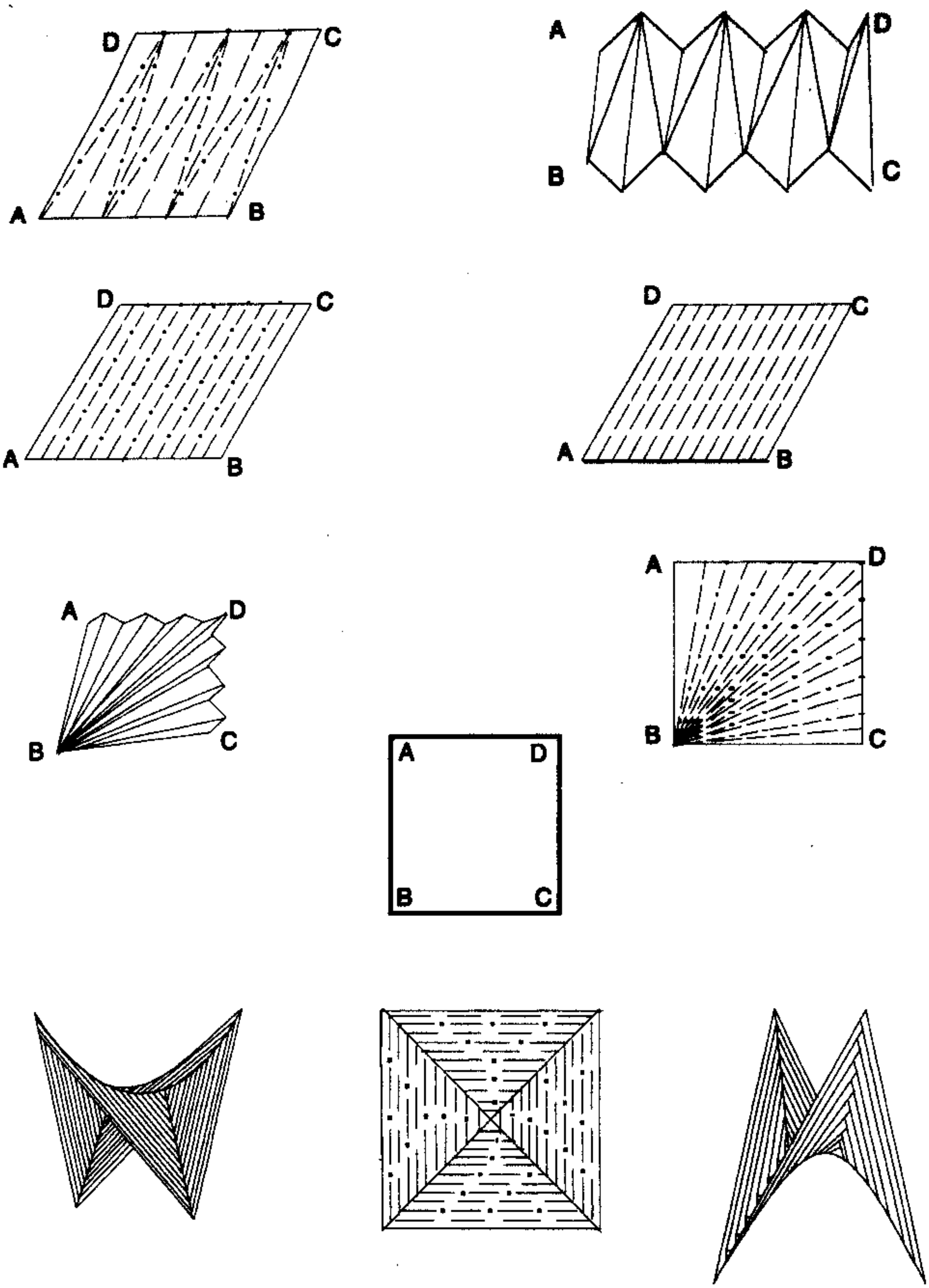


Always fold with the inverted thumb nail



Never fold with the fingers

SO ALWAYS FOLD WITH THE BACK OF YOUR THUMB NAIL.



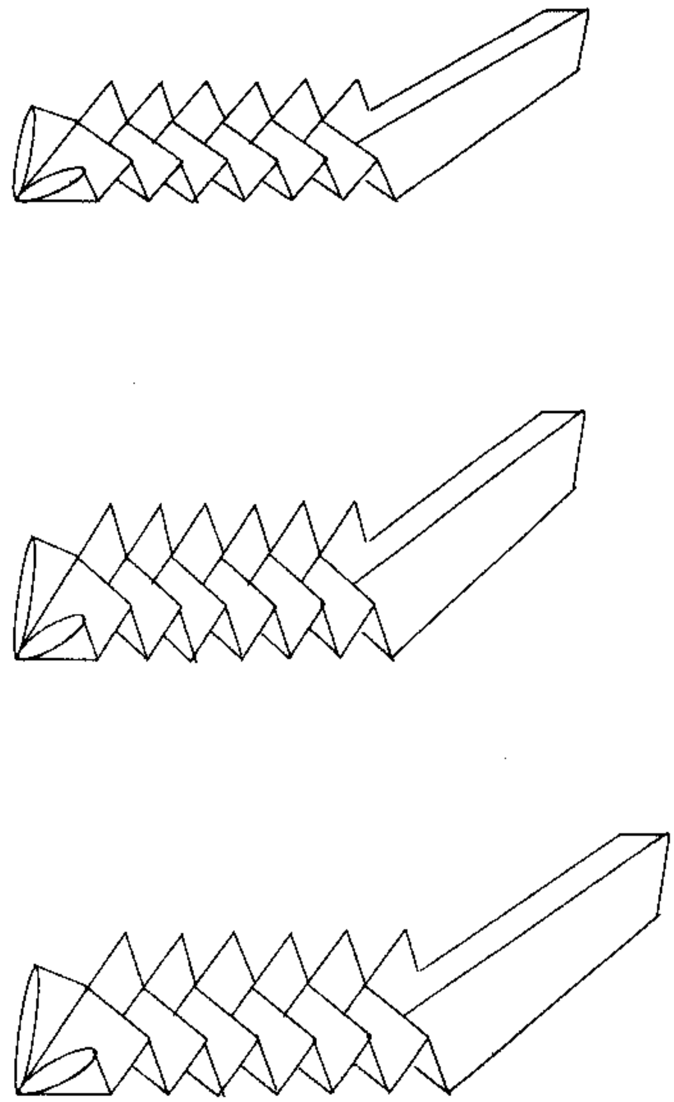
Different Designs Resulting From Repetitive Pleat Folding

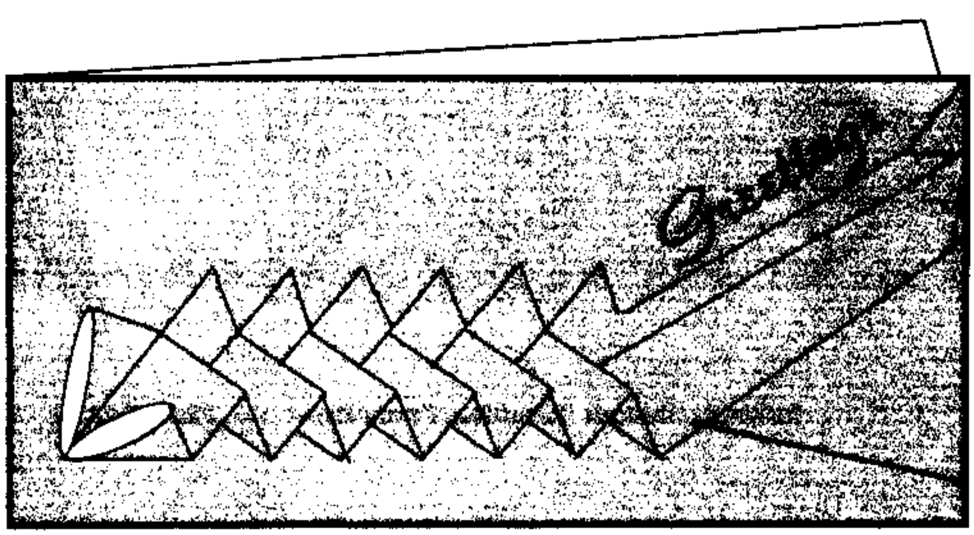
Chapter - 1
Prologue

Square paper yields to many significant shapes and curves if folded by a particular repetitive technique. These shapes are a blend of beauty and mathematics; mathematics which spans school curriculum and advance engineering mathematical complexities. Some of the shapes can be folded directly from scrap strips of paper. You can invent many designs of your own choice as an artistic flare or as training models to comprehend mathematical concepts.

From the various techniques of repetitive folding, just one is used here; a set of parallel valley folds are intersected by another set of parallel mountain folds. This technique twists the paper helically. The vertical edges of paper meander according to intersecting angles, resulting in circular shape like a rosette. Hence the model is referred as a rosette.

One of the resulting models and some variations based on it are presented in this book against the backdrop of NCERT syllabus. Of course, you can add many more to the applications list of rosettes.





Part - I
Basic

life. When we fold a square paper it is origami. When we unfold' it, it is mathematics. This reversible process with a square paper serves the purpose of mathematics laboratory with no limits. Origami is mathematics congealed into a square paper.

Some early attempts in India: In this context one recalls the singular contribution of the Indian mathematician T. Sundara Rao, whose book `Some Geometric Exercises in Paper folding' printed a century back, in 1893, blazed a new trail. It is indeed sad that this book, has never been printed in our own country.

Mathematics through origami in India: NCSTC has taken up a systematic work in popularisation of mathematics through origami since 1990. Intensive workshops on `mathematics through origami' are being held all over the country for teachers. The response amongst the teachers has been heartening as invariably there have been requests for repeat workshops. Some of the trained teachers are using this discipline amongst students. Mrs. Rajlaxmi a mathematics teacher from Hyderabad won a British Casmay Award for the same.

Theme of the Present Book: The exclusive theme of the present book is the Rosette. Rosette, a good looking mathematically "innocent" disc like model. The Rosette has been simultaneously stretched in two directions, to both explore its innate mathematical properties as well as its non mathematical beauty.

A square is big enough for every body.

Happy Folding

Ravindra Keskar
June, 2000

Preface

Nature ! Out of the simplest matter it creates most diverse things, without the slightest effort, with the greatest perfection, and on everything it casts sort of fine veil. Each of its creation has its own essence, each phenomenon has separate concept, but everything is a single whole.

Göthe

Nature and Mathematics

While creating these diverse things nature has got to organise the simplest matter in a very "particular way". This very "particular way" in essence, is the embodiment of mathematics. The mathematics of creation and beauty, of growth and entropy. Thus mathematics is the basis of all diverse and beautiful things. But nature is very illusive about its mathematics.

Man & Mathematics

Early mathematics: Early mathematics evolved from the work of the tailor and the tinkerer - all practical craftsmen. Mathematics has deep roots in practice. The very vocabulary of mathematics is replete with associations of its pragmatic past. Consider, for instance, the word "straight line". It comes from the Latin "Stretched Linen". As any farmer wanting to grow potatoes would simply stretch a string to help him sow in a straight line. Any mason would simply stretch a piece of linen to enable him to lay bricks in a straight line. So over time "Stretched Linen" became "Straight line". The "digits" 1 to 10, which we use so commonly comes from the Latin word for fingers -- the ten fingers of our hands.

School Mathematics: Today school mathematics is totally cut off from real life. The entire curriculum seems to be overlaid by "mumbo-jumbo" of professional mathematicians. In this process the entire delight and beauty of mathematics has got buried. Today, any school worth its salt, would boast of a physics, chemistry and biology laboratory — all agleam with the most modern gadgetry. Rare is a school which has even thought of having a mathematics lab. It is thought that mathematics is best learnt by mugging up tables and by repeatedly solving boring sums. Most of the children are scared of mathematics and carry (or drop-out) the burden of it through precious years in school. If children are to appreciate the beauty of mathematics, it is imperative, that they get a feel for mathematics through practical work.

Origami and Mathematics : The Japanese art of Origami (Ori to fold; Gami for paper), literally means paper folding. A single fold in a paper generates a straight line. By systematically folding a paper, one could fold lots of angles, polygons, curves and 3-D polyhedra. In this process one can learn a lot of concrete mathematics.

Nature, Origami and Mathematics

Origami is analog of life process.

Peter Engle

We simulate nature through origami. Complex animals like lobsters can be created from a single, simple square paper. Here mathematics comes alive with beauty and buoyance, with rhythm and excitement. Not in its usual frightening and faceless form of mere numbers and equations. One starts wondering about 'looks' of $\sqrt{2}$, about quadratic equation $ax^2+bx+c=0$ related to our practical

Foreword

For many kids in our schools, mathematics is a dreaded subject, perhaps even more so than physics, chemistry and biology. One reason for this, it is believed, is that whatever is taught in the name of these subjects has little or nothing to do with real life. Experience in the field has shown that once a clear link can be established, the teaching and learning of the subject at once becomes agreeable and enjoyable, the dread dissolves and children start loving it. That is why this particular aspect has been consistently kept in view in all NCSTC programmes and activities aimed at popularisation of science and mathematics.

NCSTC has been running an origami programme for nearly a decade involving training and demonstrations for teachers and activists of voluntary organisations, with Shri Ravindra Keskar as our key resource person. The programme has been very popular and fruitful. It has been made possible only through the dedication, commitment and high level of enthusiasm brought to bear on it by Shri Keskar.

This volume put together by Shri Keskar has been in the making for many years and should have come out long ago! But happily, it is never too late for such a book to come out. In any case there aren't very many other books on this subject flooding the book-stores. Even so, there is something very unique about this book. In conceiving its contents, a conscious attempt has been made to specifically relate them wherever possible to the contents of the NCERT mathematics text books of class VIII, IX & X, . This way the utility of this book stands enahnced. Should mathematics teachers teaching those classes use origami in this book to supplement their teaching, children would love it, enjoy their classes and probably end up with much better understanding of and grounding in the concepts covered.

In some ways origami can be considered as a stepping stone towards appreciation and building up of mathematics laboratories in schools. We intend bringing out more such books. We will follow this up with a reprint of Shri T. Sundara Row's book "Some Geometric Excercises in Paper-folidng" first published in 1893 in USA—reportedly never brought out in India.

The usefulness of this volume can only be gauged through the response we receive from students, teachers and other who make use of this book. We would love to hear from them, particularly with a view to improving it in its future versions. We would be happy and consider our purpose served if, because of this book, the dread of mathematics can be lowered and eliminated even among some children.

Narender K. Sehgal
Scientist 'G' & Head NCSTC and
Director, Vigyan Prasar

New Delhi
June 2000

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Mathematics Through Origami

Square Pegs in Round Holes

Ravindra Keskar

Vigyan Prasar